1. (40 points) Frequency response of the 2nd-order loop filter: Consider the loop filter in Figure 1.

(a) Figure 1 is a mix of time-domain and Z-domain symbols. Let $E(z)$, $Q(z)$, and $V(z)$ be the Z-transforms of $e(n)$, $q(n)$, and $v(n)$, respectively. Compute analytically in the Z-transform domain the filter response, $F(z) = \frac{V(z)}{E(z)}$. Solution should be in terms of $K_1$ and $K_2$, but not in terms of $Q(z)$.

(b) For $\zeta = 1$ and $B_n T_s = 0.05$, and $K_0 = -1$ and $K_p = 5.4$, calculate the coefficients $K_1$ and $K_2$ for the discrete-time proportional-plus-integrator loop filter, as described in the lecture notes, or in the Rice book page 907 (Appendix C), equation (C.58).

(c) Use Matlab’s `freqz` command to plot the amplitude and phase of the frequency response of this loop filter for the values computed in (b).

(d) Does its frequency response show it to be a combination of integrator and proportional terms? Explain why or why not.

2. (20 points) Numerically-controlled oscillator (NCO): Figure 2 shows an example sequence of the NCO. Each time a $NCO(n)$ sample is below zero, one is immediately added to it. When one is added to it, we calculate the fractional offset $0 \leq \mu < 1$ to be the fractional distance after sample time $n - 1$ to the point when the NCO signal crosses zero. Using the geometrical properties of similar triangles, show that

$$\mu = \frac{NCO(n-1)}{W(n)}$$