7e:
We need to compute $rS$, where the multiplication is “multiply and then sum mod 2”. (i) $r = [1, 1, 1, 1, 0, 0, 0]$ so

$$rS = [1, 1, 1, 1, 0, 0, 0] \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} = [3, 3, 3] \mod 2 = [1, 1, 1]$$

Since the product is not $[0, 0, 0]$, there was an error in the received vector. This vector $[1, 1, 1]$ is the first row is $S$ so the error is in the first bit. The transmitted bits were thus the first four bits but with bit 1 reversed: $[0, 1, 1, 1]$. (ii) $r = [0, 0, 0, 1, 1, 1, 0]$, so

$$rS = [0, 0, 0, 1, 1, 1, 0] \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} = [2, 2, 0] \mod 2 = [0, 0, 0]$$

Since the product is all zeros, there was no error. Thus the transmitted bits were the first four bits: $[0, 0, 0, 1]$. (iii) $r = [0, 0, 1, 1, 1, 1, 1]$, so

$$rS = [0, 0, 1, 1, 1, 1, 1] \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} = [3, 2, 2] \mod 2 = [1, 0, 0]$$

Since the product is not zeros, there was an error. The error code $[1, 0, 0]$ is the same as row #5 in $S$. Thus the received vector should have been $[0, 0, 1, 1, 0, 1, 1]$. But the first four bits were correct, and thus we can say that the transmitted bits were thus $[0, 0, 1, 1]$. 


The transmitted coded bit vector is $c = dG$, with

\[
G = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\]

(i) Encoding the data bits $d = [1, 1, 1]$, I get that

\[
c = [1, 1, 1] \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix} = [1, 1, 1, 2, 2, 2] \mod 2 = [1, 1, 1, 0, 0, 0].
\]

(ii) Encoding the data bits $d = [0, 0, 1]$, I get that

\[
c = [0, 0, 1] \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix} = [0, 0, 1, 1, 1, 0] \mod 2 = [0, 0, 1, 1, 1, 0]
\]

(iii) Encoding the data bits $d = [1, 1, 0]$, I get that

\[
c = [1, 1, 0] \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 0
\end{bmatrix} = [1, 1, 0, 1, 1, 2] \mod 2 = [1, 1, 0, 1, 1, 0]
\]