# ECE 5325/6325: Wireless Communication Systems
# Lecture Notes, Spring 2013

**Lecture 5**

Today: (1) Path Loss Models (revisited), (2) Link Budgeting

- Reading – Today: Haykin/Moher handout (2.9-2.10) (on Canvas), Molisch Section 3.2; Thu: same.
- HW 2 deadline extended to midnight tonight.
- HW 3 will be due Tuesday at the start of class, so that we can discuss the Homework in class.
- Thursday's lecture will be entirely example link budget problems; no "new" material.
- Exam 1 is Thu, Jan 31 in class (one hour).

## 1 Path Loss Models

A universal equation (also called a link budget) for received power is:

$$P_r(\text{dBW}) = P_t(\text{dBW}) + \sum \text{dB Gains} - \sum \text{dB Losses} \tag{1}$$

Of course, a dB Gain is just (-1) times a dB Loss. So whether we include something the Gains or Losses column is just a matter of our perspective. We typically think of an antenna as being a gain; and we think of path loss as being a loss. Note that when we say "path loss", by calling it a loss, we will express it as a positive value when it causes the received power to go down. If we had called it a "path gain" (as is sometimes done), then we will express it as a negative value when it causes the received power to go down.

1. There's no particular reason I chose dBW instead of dBm for $P_r$ and $P_t$. But they must be the same, otherwise you'll have a 30 dB error!

2. *If using EIRP transmit power*, it includes $P_t(\text{dBW}) + G_t(\text{dB})$, so don't double count $G_t$ by also including it in the dB Gains sum.

3. **Gains** are typically the antenna gains, compared to isotropic antennas.

4. **Losses** include large scale path loss, or reflection losses (and diffraction, scattering, or shadowing losses, if you know these specifically),

losses due to imperfect matching in the transmitter or receiver antenna, any known small scale fading loss or "margin" (what an engineer decides needs to be included in case the fading is especially bad), etc.

Path loss models are either (1) empirical or (2) theoretical. We've already studied one empirical model, the *path loss exponent model*. Below we describe two theoretical path loss models, and revisit the path loss exponent model.

## 2 Theoretical Path Loss: Free Space

Free space is nothing – nowhere in the world do we have nothing. So why discuss it?

In the "far field" (distances many wavelengths from the antenna), the received power $P_r$ in free space at a path length $d$ is given by the "Friis Equation" as

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \tag{2}$$

where $G_t$ and $G_r$ are the transmitter and receiver antenna gains, respectively; $P_t$ is the transmit power; and $\lambda$ is the wavelength. Notes:

- Wavelength $\lambda = c/f$, where $c = 3 \times 10^8$ meters/sec is the speed of light, and $f$ is the frequency. We tend to use the center frequency for $f$, except for UWB signals, it won't really matter.
- All terms in (2) are in linear units, not dB.
- The effective isotropic radiated power (EIRP) is $P_t G_t$.
- The path loss is $L_p = \left(\frac{4\pi d}{\lambda}\right)^2$. This term is called the "free space path loss".
- The received power equation (2) is called the Friis transmission equation, named after Harald T. Friis [1].
- Free space is used for space communications systems, or radio astronomy. Not for cellular telephony.

In dB, the expression from (2) becomes

$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t(\text{dB}) + G_r(\text{dB}) - L_p(\text{dB}), \quad \text{where } L_p(\text{dB}) = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) \tag{3}$$

I like to leave $L_p(\text{dB})$ in terms of $d/\lambda$, which is a unitless ratio of how many wavelengths the signal has traveled. The terms $G_t(\text{dB})$ and $G_r(\text{dB})$ are clearly gains, when they are positive, the received power increases. And as distance increases, $L_p(\text{dB})$ increases, which because of the negative sign, reduces the received power.

### 2.1 Received Power Reference

Note either (2) or (3) can be converted to "refer to" a reference distance. For example, multiply the top and bottom of (2) by $(d_0/d_0)^2$ for some reference distance $d_0$:

$$\begin{aligned} P_r &= P_t G_t G_r \left(\frac{\lambda}{4\pi d_0}\right)^2 \left(\frac{d_0}{d}\right)^2 \\ &= P_0 \left(\frac{d_0}{d}\right)^2 \end{aligned} \tag{4}$$

where $P_0 = P_t G_t G_r \left(\frac{\lambda}{4\pi d_0}\right)^2$ is the received power at the reference distance $d_0$, calculated by using (2) with the reference distance $d_0$. Now, we see that whatever the received power in free space is at distance $d_0$, the power at $d$ decays as $(d_0/d)^2$ beyond that distance. In dB terms,

$$P_r(\text{dBm}) = P_0(\text{dBm}) - 20 \log_{10} \frac{d}{d_0} \tag{5}$$

where $P_0(\text{dBm}) = 10 \log_{10} P_0$. Not only is (5) simpler than (3), it is easier to deal with in practice when you can measure $P_0(\text{dBm})$. This is useful when the antenna gains and mismatches and transmit power are not known.

### 2.2 Antennas

Antenna gain is a function of angle. The only exception is the (mythical) isotropic radiator.

**Def'n:** *Isotropic Radiator*
An antenna that radiates equally in all directions. In other words, the antenna gain $G$ is 1 (linear terms) or 0 dB in all directions.

(From Prof. Furse) An isotropic radiator must be infinitesimally small. Does not exist in practice, but is a good starting point.

Antenna gains can be referred to other ideal antenna types:

- dBi: Gain compared to isotropic radiator. Same as the dB gain we mentioned above because the isotropic radiator has a gain of 1 (or 0 dB).
- dBd: Gain compared to a half-wave dipole antenna. The 1/2 wave dipole has gain 1.64 (linear) or 2.15 dB, so dBi is 2.15 dB greater than dBd.

Technically, any antenna that is not isotropic is *directive*. Directivity is measured in the far field from an antenna as:

$$D = \frac{P_r(\text{maximum})}{P_r(\text{isotropic})}$$

where $P_r$(maximum) is the maximum received power (at the same distance but max across angle), and $P_r$(isotropic) is the power that would have been received at that point if the antenna was an isotropic radiator.

Commonly, we call an antenna *directional* if it is has a non-uniform horizontal pattern. A dipole has a "donut-shaped" pattern, which is a circle in its horizontal pattern (slice).

There are also antenna mismatches. We denote these as $\Gamma_t$ and $\Gamma_r$. Both are $\leq 1$, and only one if there is a perfect impedance match and no loss.

### 2.3 Theoretical: Exponential Decay

In some propagation environments, power decays exponentially due to its propagation through the environment. Compared to the free-space propagation equation, which says that waves propagate in a vacuum, the equation changes to include a term proportional to $10^{\alpha d/10}$, where $\alpha$ is a loss factor, with units dB per meter. In this case, $L_p(dB) = \alpha d$, which makes it easier to see that $\alpha$ is dB loss per meter. Equation (2) is typically re-written as:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 10^{-\alpha d/10} \tag{6}$$

This works well in some conditions, for example, at 60 GHz, at which oxygen molecules absorb RF radiation, or due to rain at 30 GHz. In effect, because the wave is propagating through a "material" (air or water), its power is absorbed and lost (turned into heat) in the medium.

### 2.4 Path Loss Exponent Model

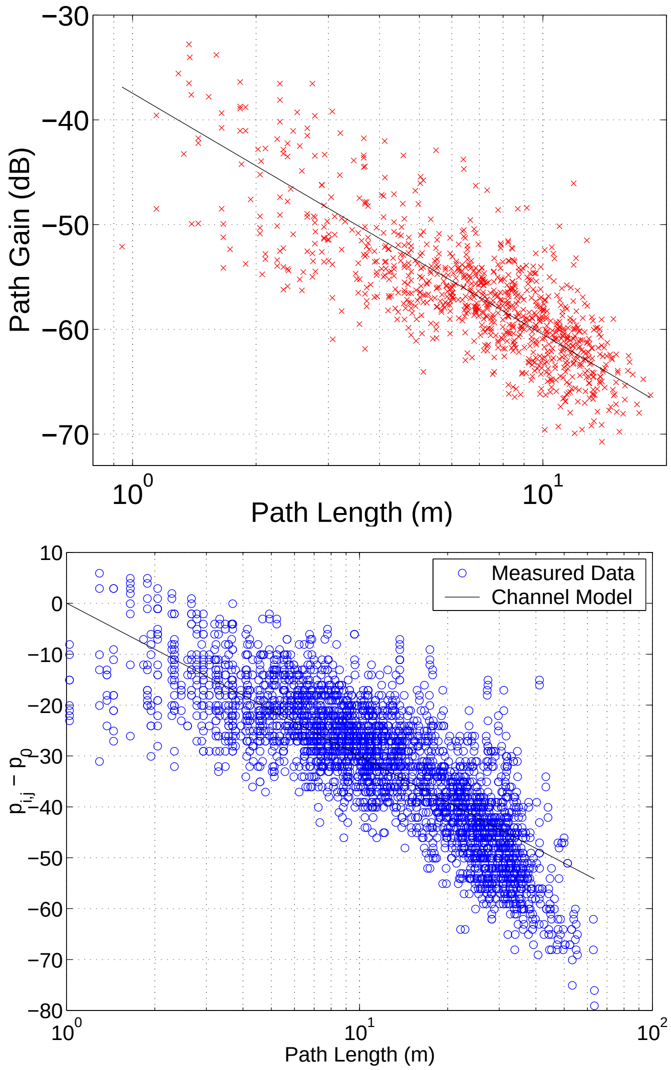
We've already presented the path loss exponent model, we is a modification of (5) as

$$P_r(\text{dBm}) = P_0(\text{dBm}) - 10n \log_{10} \frac{d}{d_0} \tag{7}$$

where $P_0$(dBm) is still given by the Friis equation, but now the $L_p$(dB) term has changed to include a factor $10n$ instead of 20. Typically $d_0$ is taken to be on the edge of near-field and far-field, say 1 meter for indoor propagation, and 10-100m for outdoor propagation.

We mentioned that we can find the parameter $P_0$(dBm) from measurements. We can also find the parameter $n$ from measurements. For example, two measurement campaigns I did in office areas resulted in the estimates of $n = 2.30$ and 2.98 as shown in Figure 1.

Empirical measurement studies sometimes show a change in slope of the $L_p$ vs. distance curve a certain distance [2]. You can see this effect in Figure 1(b) for $d > 20$ meters; the path gains at $d = 50$ meters are all lower than the model, and one can see the slope changing to an $n$ higher




(a)

(b)

Figure 1: (a) Wideband path gain measurements (x) at 2.4 GHz as a function of path length $d$. Linear fit (—) is with $d_0$ = 1m, $n$ = 2.30, and $\sigma_{dB} = 3.92$. (b) Narrowband measurements of received power minus $P_0$(dBm) (o) at 925 MHz as a function of path length $d$. Linear fit (—) is with $d_0$ = 1m, $n$ = 2.98, with standard deviation $\sigma_{dB}$ = 7.38. From [3].

than 2.98. We can model the path loss as experiencing more than one slope in different segments of the $\log d$ axis.

$$P_r(\text{dBm}) = \begin{cases} P_0(\text{dBm}) - 10 n_1 \log_{10} \frac{d}{d_0}, & d \le d_1 \\ P_1(\text{dBm}) - 10 n_2 \log_{10} \frac{d}{d_1}, & d > d_1 \end{cases} \quad (8)$$

where $P_0$(dBm) is still the Friis received power at a distance $d_0$, and $P_1$(dBm) is the received power (given by the first line of the equation) at distance $d_1$, and $d_0 < d_1$. Typically, the slope of the path loss increases, *i.e.*, $n_2 > n_1$.

# 3 Link Budgeting

Link budgets are, as the name implies, an accounting of the gains and losses that occur in a radio channel between a transmitter and receiver. We've talked about S/I – you need an acceptable signal to interference ratio. In addition, you need an acceptable signal to noise, or S/N, ratio. (a.k.a. SNR, $C/N$, or $P_r/P_N$ ratio, where $C$ stands for carrier power, the same thing we've been calling $P_r$, and $N$ or $P_N$ stands for noise power. Since we've already used $N$ in our notation for the cellular reuse factor, we denote noise power as $P_N$ instead.) Noise power is due to thermal noise.

In the second part of this course, we will provide more details on where the requirements for S/N ratio come from. For now, we assume a requirement is given. For a given required S/N ratio, some valid questions are: What is the required base station (or mobile) transmit power? What is the maximum cell radius (*i.e.*, path length)? What is the effect of changing the frequency of operation? Also, there is a concept of path balance, that is, having connectivity in only one direction doesn't help in a cellular system. So using too much power in either BS or mobile to make the maximum path length longer in one direction is wasteful.

As we've said, this is accounting. We need to keep track of each loss and each gain that is experienced. Also, to find the noise power $P_N$, we need to know the characteristics of the receiver.

## 3.1 Link Budget Procedure

A universal link budget for $S/N$ is:

$$S/N = P_r(\text{dBW}) - P_N(\text{dBW}) = P_t(\text{dBW}) + \sum \text{dB Gains} - \sum \text{dB Losses} - P_N(\text{dBW})$$

Compared to (1), we simply subtract the dB noise power.

1. The dB noise figure $F$ (dB) is either included in $P_N$(dBW) or in the dB losses, *not both*!

2. Sometimes the *receiver sensitivity* is given (for example on a RFIC spec sheet). This is the $P_N$(dB) plus the required $S/N$(dB).

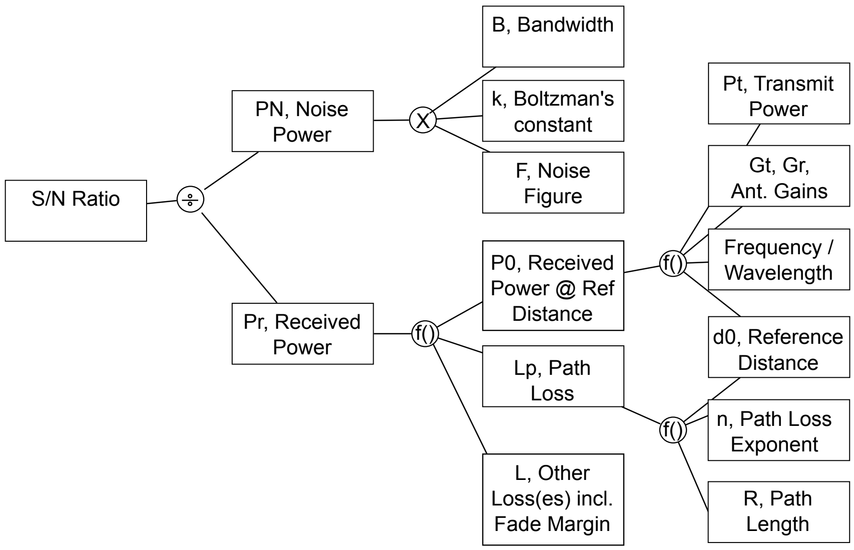



Figure 2: Relationship among link budget variables.

## 3.2 Thermal noise

The thermal noise power in the receiver is $P_N$, and is given as $P_N = FkT_0B$, where

- $k$ is Boltzmann's constant, $k = 1.38 \times 10^{-23} J/K$. The units are J/K (Joules/Kelvin) or W·s/K (1 Joule = 1 Watt × second).
- $T_0$ is the ambient temperature, typically taken to be 290-300 K. If not given, use 294 K, which is 70 degrees Fahrenheit.
- $B$ is the bandwidth, in Hz (equivalently, 1/s).
- $F$ is the (unitless) noise figure, which quantifies the gain to the noise produced in the receiver. The noise figure $F \geq 1$.

In dB terms,

$$P_N(\text{dBW}) = F(\text{dB}) + k(\text{dBWs/K}) + T_0(\text{dBK}) + B(\text{dBHz})$$

where $k(\text{dBWs/K}) = 10 \log_{10} 1.38 \times 10^{-23}\text{J/K} = -228.6$ dBWs/K. We can also find $F$ from what is called the equivalent temperature $T_e$. This is sometimes given instead of the noise figure directly.

$$F = 1 + \frac{T_e}{T_0}$$

**Example: Sensor Network**

Assume two wireless sensors 1 foot above ground need to communicate over a range of 30 meters. They operate the 802.15.4 standard

(DS-SS at 2.4 GHz). Assume the log-distance model with reference distance 1m, with path loss at 1 m is $\Pi_0 = 40$ dB, and path loss exponent 3 beyond 1m. Assume the antenna gains are both 3.0 dBi. The transmitter is the TI CC2520, which has max $P_t = 1$ mW, and its spec sheet gives a receiver sensitivity of -98 dBm. What is the fading margin at a 30 meter range? (Note: By the end of lecture 10 you will be able to specify fading margin given a desired probability that communication will be lost due to a severe fade. Here we're just finding what the margin is for these system parameters.)

## References

[1] J. E. Brittain. Electrical engineering hall of fame: Harald T. Friis. *Proceedings of the IEEE*, 97(9):1651–1654, Sept. 2009.

[2] M. Feuerstein, K. Blackard, T. Rappaport, S. Seidel, and H. Xia. Path loss, delay spread, and outage models as functions of antenna height for microcellular system design. *Vehicular Technology, IEEE Transactions on*, 43(3):487–498, Aug 1994.

[3] N. Patwari. *Location Estimation in Sensor Networks*. PhD thesis, University of Michigan, Ann Arbor, MI, Sept. 2005.