

ECE 5325/6325: Wireless Communication Systems

Lecture Notes, Spring 2010

Lecture 5

Today: (1) Free Space (2) Large Scale Path Loss (Intro)

- Reading for today's lecture: 4.1-4.3, 4.11. For Thu: 4.4-4.5.
- Homework: Please number questions as numbered on assignment, and turn in solution pages in order.

Path loss models are either (1) empirical or (2) theoretical. We'll start to discuss both. As you'll see, empirical models were developed as modifications to theoretical models.

1 Free Space Propagation

Free space is nothing. So why discuss it?

Section 4.3 describes the electric and magnetic fields produced by a small dipole antenna. This equation is only valid for a small dipole, and is only useful very close to (the near field of) the antenna. In the "far field" (distances many wavelengths from the antenna), the received power P_r in free space at a path length d is given in Section 4.2 as

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \quad (1)$$

where G_t and G_r are the transmitter and receiver antenna gains, respectively; P_t is the transmit power; and λ is the wavelength. Notes:

- Wavelength $\lambda = c/f$, where $c = 3 \times 10^8$ meters/sec is the speed of light, and f is the frequency. We tend to use the center frequency for f , except for UWB signals, it won't really matter.
- All terms in (1) are in linear units, not dB.
- The effective isotropic radiated power (EIRP) is $P_t G_t$.
- The path loss is $L_p = \left(\frac{\lambda}{4\pi d} \right)^2$. This term is called the "free space path loss".
- The received power equation (1) is called the Friis transmission equation, named after Harald T. Friis (from Wikipedia).

- Free space is used for space communications systems, or radio astronomy. Not for cellular telephony.

In dB, the expression from (1) becomes

$$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t(\text{dB}) + G_r(\text{dB}) - L_p(\text{dB}), \quad \text{where } L_p(\text{dB}) = 20 \log_{10} \left(\frac{4\pi d}{\lambda} \right) \quad (2)$$

I like to leave $L_p(\text{dB})$ in terms of d/λ , which is a unitless ratio of how many wavelengths the signal has traveled. The terms $G_t(\text{dB})$ and $G_r(\text{dB})$ are clearly gains, when they are positive, the received power increases. And as distance increases, $L_p(\text{dB})$ increases, which because of the negative sign, reduces the received power. *We use “G” to denote gains and “L” to denote losses.* But a negative gain is a loss, and a negative loss is a gain.

1.1 Received Power Reference

Note either (1) or (2) can be converted to “refer to” a reference distance. For example, multiply the top and bottom of (1) by $(d_0/d)^2$ for some reference distance d_0 :

$$\begin{aligned} P_r &= P_t G_t G_r \left(\frac{\lambda}{4\pi d_0} \right)^2 \left(\frac{d_0}{d} \right)^2 \\ &= P_r(d_0) \left(\frac{d_0}{d} \right)^2 \end{aligned} \quad (3)$$

where $P_r(d_0)$ is the received power at the reference distance d_0 , according to (1). Now, we see that whatever the received power in free space is at distance d_0 , the power at d decays as $(d_0/d)^2$ beyond that distance. In dB terms,

$$P_r(\text{dBm}) = \Pi_0(\text{dBm}) - 20 \log_{10} \frac{d}{d_0} \quad (4)$$

where $\Pi_0(\text{dBm}) = 10 \log_{10} P_r(d_0)$. This is actually an easier equation to deal with in practice, because we don’t necessarily know the antenna gains and mismatches, and transmit power; but we can measure $\Pi_0(\text{dBm})$. Of course, not in free space – we don’t exist there!

1.2 Antennas

Antenna gain is a function of angle. The only exception is the (mythical) isotropic radiator.

Def’n: *Isotropic Radiator*

An antenna that radiates equally in all directions. In other words, the antenna gain G is 1 (linear terms) or 0 dB in all directions.

(From Prof. Furse) An isotropic radiator must be infinitesimally small. Does not exist in practice, but is a good starting point.

Antenna gains can be referred to other ideal antenna types:

- dBi: Gain compared to isotropic radiator. Same as the dB gain we mentioned above because the isotropic radiator has a gain of 1 or 0 dB.
- dBd: Gain compared to a half-wave dipole antenna. The 1/2 wave dipole has gain 1.64 (linear) or 2.15 dB, so dBi is 2.15 dB greater than dBd.

Technically, any antenna that is not isotropic is *directive*. Directivity is measured in the far field from an antenna as:

$$D = \frac{P_r(\text{maximum})}{P_r(\text{isotropic})}$$

where $P_r(\text{maximum})$ is the maximum received power (at the same distance but max across angle), and $P_r(\text{isotropic})$ is the power that would have been received at that point if the antenna was an isotropic radiator.

Antennas also have an efficiency. They lose some power without radiating it as EM waves. Thus the maximum gain is the directivity times the efficiency.

Commonly, we call an antenna *directional* if it has a non-uniform horizontal pattern. A dipole has a “donut-shaped” pattern, which is a circle in its horizontal pattern (slice).

There are also antenna mismatches. We denote these as Γ_t and Γ_r . Both are ≤ 1 , and only one if there is a perfect impedance match and no loss.

1.3 Power Flux Density

There is a concept in propagation of power flux density, the amount of power that travels through a given area. This is a far-field concept only. Power flux is denoted P_d in Rappaport, and has units of Watts per square meter, W/m^2 . In free space,

$$P_d = \frac{|E|^2}{\eta} \text{W}/\text{m}^2$$

where η is the intrinsic impedance of free space, $120\pi\Omega = 377\Omega$, and $|E|^2$ is the magnitude squared of the electric field. The idea is that an antenna “captures” some of this power, according to, effectively, how large the antenna is. We call this the *effective antenna aperture*, and denote it A_e , with units m^2 . In short, physically larger antennas are capable of larger A_e , although there is no exact proportionality. In this case the definition of the received power is

$$P_r(d) = P_d A_e$$

2 Large Scale Path Loss Models

Let's transition to the real world, where we exist. There are other effects besides radiation, including attenuation (transmission), reflection, diffraction, scattering, etc. We will discuss each of these in upcoming lectures. For now, suffice it to say that many signals arrive at the receiver, but with less power than would be indicated by the Friis equation. The received power varies strongly (5-25 dB) even for small changes in antenna position, center frequency, and time. But, there is a large effect caused when the distance (a.k.a. path length) increases by orders of magnitude. This large effect we call *large scale path loss*. It is analogous to L_p , but doesn't necessarily take the same form. We will re-write L_p as a function of distance in one of two ways:

1. Exponential decay: L_p will include a term proportional to $10^{\alpha d/10}$, where α is a loss factor, with units dB per meter. In this case, $L_p(\text{dB}) = \alpha d$, which makes it easier to see that α is dB loss per meter. Equation (1) is typically re-written as:

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 10^{\alpha d/10} \quad (5)$$

This works well in some conditions, for example, at 60 GHz, at which oxygen molecules absorb RF radiation, or due to rain at 30 GHz.

2. Power decay: L_p will be proportional to $1/d^n$, for some path loss exponent n . In free space, it was proportional to $1/d^2$, so this just lets n adjust to the particular environment. Typically, n ranges between 1.6 and 6, according to Rappaport. From my experience, I've seen n between 1.7 and 5.

2.1 Log Distance Path Loss

This is 4.11.3 in Rappaport. This is synonymous with what I call "power decay" above. Actually, it is the simplest of the models, and makes a big step towards better representation of actual large-scale path loss. In the log-distance path loss model, we can simply write the received power as a modification of (4) as

$$P_r(\text{dBm}) = \Pi_0(\text{dBm}) - 10n \log_{10} \frac{d}{d_0} \quad (6)$$

where $\Pi_0(\text{dBm})$ is still given by the Friis equation, but now the $L_p(\text{dB})$ term has changed to include a factor $10n$ instead of 20. Typically d_0 is taken to be on the edge of near-field and far-field, say 1 meter for indoor propagation, and 10-100m for outdoor propagation.

We mentioned that we can find the parameter $\Pi_0(\text{dBm})$ from measurements. We can also find the parameter n from measurements. For example, two measurement campaigns I did in office areas resulted in the estimates of $n = 2.30$ and 2.98 as shown in Figures 1 and 2.

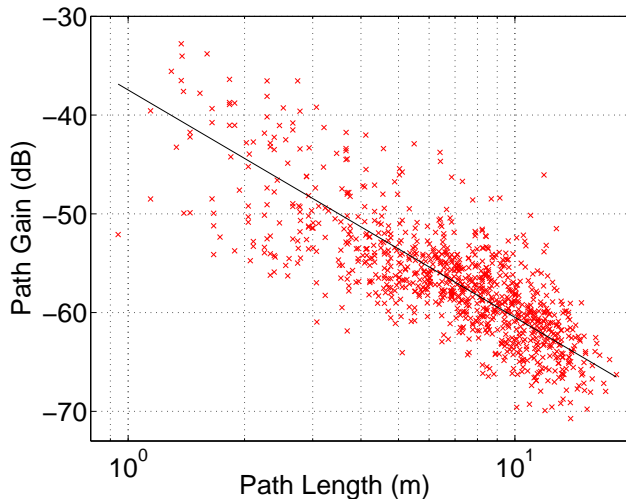


Figure 1: Wideband path gain measurements (x) at 2.4 GHz as a function of path length d . Linear fit (—) is with $d_0 = 1\text{m}$, $n = 2.30$, and $\sigma_{dB} = 3.92$. From [2].

2.2 Multiple Breakpoint Model

This is Rappaport Section 4.11.4. Empirically measurement studies have shown that the slope of the L_p vs. distance curve changes after a certain distance [1]. You can see this effect in Figure 2 for $d > 20$ meters; the path gains at $d = 50$ meters are all lower than the model, and one can see the slope changing to an n higher than 2.98. We will discuss theoretical reasons why this might happen in later lectures. Regardless, we can model the path loss as experiencing more than one slope in different segments of the $\log d$ axis.

$$P_r(\text{dBm}) = \begin{cases} \Pi_0(\text{dBm}) - 10n_1 \log_{10} \frac{d}{d_0}, & d \leq d_1 \\ \Pi_1(\text{dBm}) - 10n_2 \log_{10} \frac{d}{d_1}, & d > d_1 \end{cases} \quad (7)$$

where $\Pi_0(\text{dBm})$ is still the Friis received power at a distance d_0 , and $\Pi_1(\text{dBm})$ is the received power (given by the first line of the equation) at distance d_1 , and $d_0 < d_1$. Typically, the slope of the path loss increases, *i.e.*, $n_2 > n_1$.

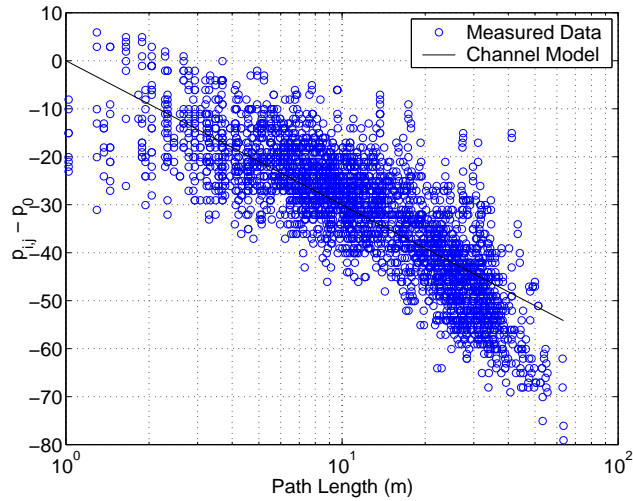


Figure 2: Narrowband measurements of path gain minus Π_0 (dBm) (o) at 925 MHz as a function of path length d . Linear fit (—) is with $d_0 = 1\text{m}$, $n = 2.98$, with standard deviation $\sigma_{dB} = 7.38$. From [2].

References

- [1] M. Feuerstein, K. Blackard, T. Rappaport, S. Seidel, and H. Xia. Path loss, delay spread, and outage models as functions of antenna height for microcellular system design. *Vehicular Technology, IEEE Transactions on*, 43(3):487–498, Aug 1994.
- [2] N. Patwari. *Location Estimation in Sensor Networks*. PhD thesis, University of Michigan, Ann Arbor, MI, Sept. 2005.