ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 8

Today: (1) Reflection / transmission, (2) Diffraction, and (3) Scattering

- Reading: Today – Molisch 4.2-4.3, and MUSE video. Thursday:

1 Reflection and Transmission

There are electric and magnetic waves that serve to propagate radio energy. The electric waves can be represented as a sum of two orthogonal *polarization* components, for example, vertical and horizontal, or left-hand and right-hand circular. What happens when these two components of the electric field hit the boundary between two different dielectric media?

We talk about the plane of incidence, that is, the plane containing the direction of travel of the waves (incident, reflected, and transmitted), and perpendicular to the surface (plane where the two media meet). See Figure 4.1 from the Molisch book, reproduced in Fig 1.

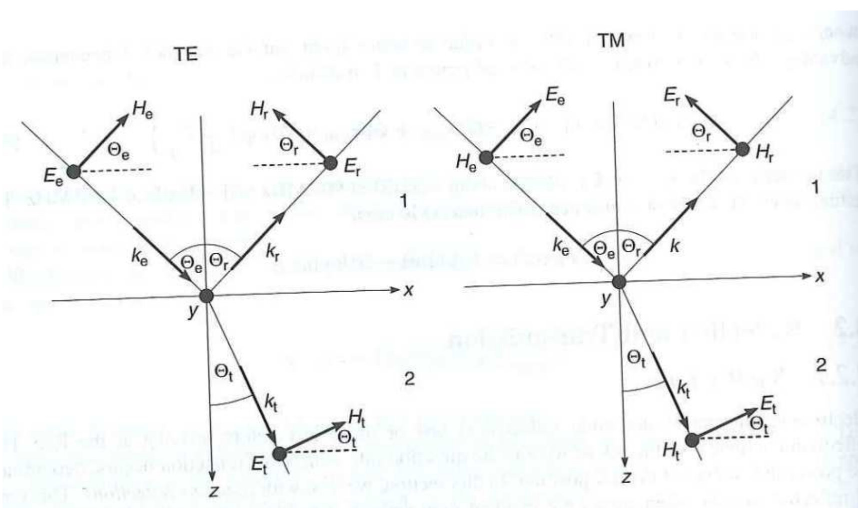

Figure 4.1 Reflection and transmission.

Figure 1: Figure 4.1 from Molisch. The solid circle indicates an arrow pointing out of the paper.

Polarization: The Molisch book denotes polarizations as "Transverse Magnetic", that is, with magnetic (H) field perpendicular to the plane of incidence, and "Transverse Electric", that is, with electric (E) field perpendicular to the plane of incidence.

A review of the basic EM parameters:

- ϵ_k is the permittivity of medium k. (units Farads/meter) (Note: F = sec / Ω) Note that here ϵ_k is often written as $\epsilon_k = \epsilon_{r,k}\epsilon_0$, where $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is free space permittivity and $\epsilon_{r,k}$ is the relative permittivity of medium k. Don't get confused by the subscripts in this equation – ϵ_r is the "name" of the variable, and k is the number of the medium.

- μ_k is the permeability of medium k. (units Henries/meter) (Note: H = Ω sec) Here, we generally take $\mu_k = 1$ H/m, which is true for most materials waves will be interacting with.

- σ_k is the conductance of medium k (units Siemens/meter). (Note: S = 1 / Ω)

- δ_k is the complex permittivity of medium k, given by $\delta_k = \epsilon_k - j\frac{\sigma_k}{2\pi f}$, where f is the wave frequency. A material is considered a good conductor when $\sigma_k > f\epsilon_k$. When a material is not a good conductor, we can ignore the imaginary part of the complex permittivity.

Use E_e, E_r, and E_t to refer to the excitation (incident), reflected, and transmitted field. These make angle with the boundary Θ_e, Θ_r, and Θ_t, respectively. We want to know the ratio of the reflected field to the incident field, *i.e.*, $\Gamma = E_r/E_e$. Each polarization has its own different reflection or transmission coefficient, so there is Γ_{TM} and Γ_{TE}. We also want to know what Θ_r and Θ_t are.

This solution comes from solving Maxwell's equations with a plane boundary between two infinite media. What Maxwell's equations result in are the following:

$$\Theta_e = \Theta_r \tag{1}$$

$$E_r = \Gamma E_e \tag{2}$$

$$E_t = (1+\Gamma)E_e \tag{3}$$

where you chose Γ based on the polarization of the incident E-field, *i.e.*, use either Γ_{TM} or Γ_{TE}, given by:

$$\begin{aligned} \Gamma_{TM} \triangleq \frac{E_r}{E_e} &= \frac{\sqrt{\delta_2}\cos\Theta_e - \sqrt{\delta_1}\cos\Theta_t}{\sqrt{\delta_2}\cos\Theta_e + \sqrt{\delta_1}\cos\Theta_t} \\ \Gamma_{TE} \triangleq \frac{E_r}{E_e} &= \frac{\sqrt{\delta_1}\cos\Theta_e - \sqrt{\delta_2}\cos\Theta_t}{\sqrt{\delta_1}\cos\Theta_e + \sqrt{\delta_2}\cos\Theta_t} \end{aligned} \tag{4}$$

The angle of transmission is determined by *Snell's Law*:

$$\sqrt{\delta_1}\sin\Theta_e = \sqrt{\delta_2}\sin\Theta_t$$

Note that the Molisch book uses ρ where I use Γ for the reflection coefficient.

Example: Simplify the reflection coefficient expressions when the first medium is air and the 2nd medium is not a good conductor.
Note that air is approximately free space (in terms of EM parameters), thus $\epsilon_1 = \epsilon_0$. Let $\epsilon_2 = \epsilon_r\epsilon_0$.

$$\begin{aligned}\Gamma_{TM} &= \frac{\sqrt{\epsilon_r\epsilon_0}\cos\Theta_e - \sqrt{\epsilon_0}\cos\Theta_t}{\sqrt{\epsilon_r\epsilon_0}\cos\Theta_e + \sqrt{\epsilon_0}\cos\Theta_t} \\ \Gamma_{TM} &= \frac{\sqrt{\epsilon_r}\cos\Theta_e - \cos\Theta_t}{\sqrt{\epsilon_r}\cos\Theta_e + \cos\Theta_t}\end{aligned} \tag{5}$$

This answer is fine, but it requires that we separately calculate Θ_t from Snell's Law. In this case, though, we can simplify Snell's Law,

$$\begin{aligned}\sin\Theta_e &= \sqrt{\epsilon_r}\sin\Theta_t \\ \frac{1}{\sqrt{\epsilon_r}}\sin\Theta_e &= \sin\Theta_t\end{aligned}$$

Finally, replacing $\cos\Theta_t = \sqrt{1-\sin^2\Theta_t} = \sqrt{1-(\sin^2\Theta_e)/\epsilon_r}$, and multiplying top and bottom by $\sqrt{\epsilon_r}$,

$$\Gamma_{TM} = \frac{\epsilon_r\cos\Theta_e - \sqrt{\epsilon_r - \sin^2\Theta_e}}{\epsilon_r\cos\Theta_e + \sqrt{\epsilon_r - \sin^2\Theta_e}} \tag{6}$$

Similarly, the same math would get you to:

$$\begin{aligned}\Gamma_{TE} &= \frac{\cos\Theta_e - \sqrt{\epsilon_r}\cos\Theta_t}{\cos\Theta_e + \sqrt{\epsilon_r}\cos\Theta_t} \\ &= \frac{\cos\Theta_e - \sqrt{\epsilon_r}\sqrt{1-(\sin^2\Theta_e)/\epsilon_r}}{\cos\Theta_e + \sqrt{\epsilon_r}\sqrt{1-(\sin^2\Theta_e)/\epsilon_r}} \\ &= \frac{\cos\Theta_e - \sqrt{\epsilon_r - \sin^2\Theta_e}}{\cos\Theta_e + \sqrt{\epsilon_r - \sin^2\Theta_e}}\end{aligned}$$

See Figure 4.2 on page 51 of the Molisch book. At some angle Θ_e, there is no reflection of the TM-polarized field. This angle is called the "Brewster angle".

Example: Reflection from ground
Find the reflection coefficients for typical ground at $\Theta_e = 75$ degrees at 100 MHz.

2 Diffraction

In EM wave propagation *Huygens' principle* says that at each point, the wave field is effectively re-radiating in all directions. In free space, these *secondary reradiators* sum and "produce" the effect of a wave front advancing in the direction away from the source. When objects exist in free space that block or attenuate some of the wave field, the reradiation enable EM waves to "bend" around objects. In order to calculate the field at a point in (or near) the "shadow" of an object, we can use Huygens' principle to find accurate numerical results. This is a short version of some advanced electromagnetics. See [2] Chapter 9 for a more detailed treatment.

See Figure 4.6 in Molisch. The Fresnel-Kirchoff parameter ν is given by,

$$\nu = h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} \tag{7}$$

depends on the geometry and the frequency, and is unitless:

- d_1, d_2, distance along line-of-sight path from TX or RX to obstruction
- h, screening height

In short, we have a normalized vertical axis at the knife edge. The top of the knife edge is at position ν – below ν, there is a perfect conductor, and above ν, there is free space. We assume the knife edge is infinitely narrow. For our point of interest beyond the knife edge, Huygens' principle has us consider (sum) the effect of the secondary reradiators along the vertical axis, above the knife edge. The summation is actually an integral, and is taken from ν to ∞, and is called the complex Fresnel integral,

$$F(\nu) = \frac{1+j}{2}\int_{\nu}^{\infty} \exp\left[-\frac{j\pi t^2}{2}\right] dt \tag{8}$$

One can calculate this integral with some knowledge of complex analysis, or even Matlab, but there is no easy analytical function that comes out of the solution. We typically use a table or a plot. The dB magnitude of the power loss from the Fresnel integral, $20\log_{10} F(\nu)$, which we call the knife-edge diffraction GAIN, is given by Figure 4.14 in the Rappaport book [1], and is copied in Figure 2. The plot is given in a linear scale in Figure 4.5 in the Molisch book, which is useful, but we really need the dB scale to get an accurate value.

Expressions exist for the multiple knife edge diffraction problem – when multiple obstructions block a propagating wave. These are detailed in Section 4.3.2 in the Molisch book. These are largely computed via numerical analysis, and we will not elaborate on them in this class.

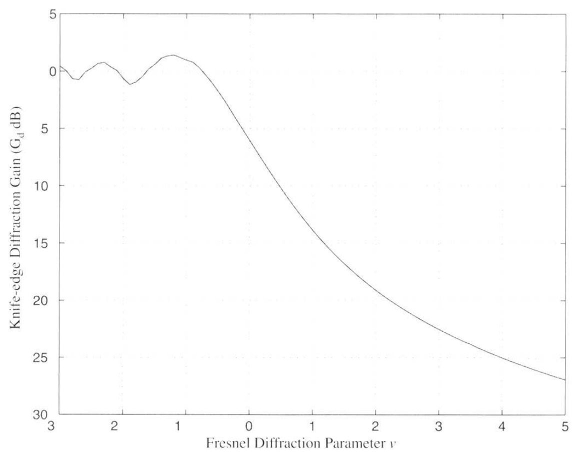

Figure 4.14 Knife-edge diffraction gain as a function of Fresnel diffraction parameter v.

Figure 2: Knife-edge diffraction gain in dB.

3 Rough Surface Scattering

When we discussed reflection, we said a wave was impinging on a flat surface. But most surfaces (*e.g.*, walls, ground) are *rough*, not flat. When is a surface considered rough? When the maximum height "protuberance" from the surface, h, is greater than h_c,

$$h_c = \frac{\lambda}{8\sin(90^o - \Theta_e)} \tag{9}$$

where Θ_e is, again, the angle of incidence.

Scattering has two effects important to us:

1. Rough surface scattering reduces the power in the reflected wave.
2. Scattering causes additional multipath to be received in directions other than the specular direction (recall $\Theta_r = \Theta_i$).

For 1., if the surface is rough, then the reflected wave has reflection coefficient multiplied by ρ_s, so that $\Gamma_{rough} = \rho_S \Gamma$. Multiple expressions exist to compute ρ_S. The approximation given in the Molisch book is:

$$\rho_S = \exp\left[-8\left(\frac{\pi\sigma_h \sin(90^o - \Theta_e)}{\lambda}\right)^2\right]$$

where σ_h is the standard deviation of the height of the rough surface. The second expression is considered to be more accurate.

For 2., scattering is a very useful and impactful phenomena. Scattering is the basis of radar, weather systems, and passive RFID tags. Engineers have put much effort into understanding the phenomena of EM wave scattering.

Similar to Huygens' principal, the wave field at the scatterer is assumed to become a secondary re-radiator. However, the object (the scatterer) is assumed to absorb the incident power, and then re-radiate it (The re-radiation is not assumed to be occurring from the free space near the object). The object is considered a new point source, where power is "received" and "retransmitted". *Essentially we have two links, one from the transmitter to the scatterer, and one away from the scatterer to the end receiver.*

Typical airplane and weather radar is monostatic, *i.e.*, the TX and RX are co-located. In some *bistatic* wireless comm systems (and more robust airplane radar systems) the TX and RX are not in the same place. The *bistatic radar equation* describes the received power P_r in a scattered wave at a receiver that is not necessarily at the transmitter location. In linear and dB terms,

$$\begin{aligned} P_r &= \frac{P_t G_t G_r \sigma_{RCS} \lambda^2}{(4\pi)^3 d_t^2 d_r^2} \\ P_r(\text{dBW}) &= P_t(\text{dBW}) + G_t(\text{dB}) + G_r(\text{dB}) + \sigma_{RCS}(\text{dBmeter}^2) \\ &\quad +20\log_{10}\lambda - 30\log_{10}4\pi - 20\log_{10}d_t - 20\log_{10}d_r \end{aligned}$$

Note that there are two $1/d^2$ terms, one corresponding to each "link" described above. The σ_{RCS} term has units of meter^2 and is the *radar cross section* of the scatterer. It is an area, like an antenna effective aperture, that describes how much power is absorbed by the scatterer to be reradiated. Also, note that in the dB expression, the dB meter^2 units cancel because there are two dB meter^2 terms on top (σ_{RCS} and $20\log_{10}\lambda^2$) and two dB meter^2 terms on bottom ($20\log_{10}d_t$ and $20\log_{10}d_r$).

References

[1] T. Rappaport. *Wireless Communications: Principles and Practice*. Prentice Hall PTR, Upper Saddle River, NJ, USA, 2nd edition, 2001.

[2] W. L. Stutzman and G. A. Theile. *Antenna Theory and Design*. John Wiley & Sons, 1981.