## Lecture 8

Today: (1) Diffraction (2) Scattering

- Reading for today's lecture: Rap 4.7, 4.8. Tue Feb 9: Exam 1.
- OH Fri 9-noon (J. Kemp), Mon 4-5, Tue 9:30-10:30, or by appt.


## 1 Diffraction

In EM wave propagation Huygens' principle says that at each point, the wave field is effectively re-radiating in all directions. In free space, these secondary reradiators sum and "produce" the effect of a wave front advancing in the direction away from the source. When objects exist in free space that block or attenuate some of the wave field, the reradiation enable EM waves to "bend" around objects. In order to calculate the field at a point in (or near) the "shadow" of an object, we can use Huygens' principle to find accurate numerical results. This is a short version of some advanced electromagnetics. See [1] Chapter 9 for a more detailed treatment.

See Figure 4.13 in Rappaport. The Fresnel-Kirchoff parameter $\nu$ is given by,

$$
\begin{equation*}
\nu=h \sqrt{\frac{2\left(d_{1}+d_{2}\right)}{\lambda d_{1} d_{2}}} \tag{1}
\end{equation*}
$$

depends on the geometry and the frequency, and is unitless:

- $d_{1}, d_{2}$, distance along line-of-sight path from TX or RX to obstruction
- $h$, screening height

In short, we have a normalized vertical axis at the knife edge. The top of the knife edge is at position $\nu$ - below $\nu$, there is a perfect conductor, and above $\nu$, there is free space. We assume the knife edge is infinitely narrow. For our point of interest beyond the knife edge, Huygens' principle has us consider (sum) the effect of the secondary reradiators along the vertical axis, above the knife edge. The summation is actually an integral, and is taken from $\nu$ to $\infty$, and is called the complex Fresnel integral,

$$
\begin{equation*}
F(\nu)=\frac{1+j}{2} \int_{\nu}^{\infty} \exp \left[-\frac{j \pi t^{2}}{2}\right] d t \tag{2}
\end{equation*}
$$

Another 6325 assignment has been posted regarding (2). The dB magnitude of the power loss from the Fresnel integral, $20 \log _{10} F(\nu)$, which we call the knife-edge diffraction GAIN, is given by Figure 4.14 in Rappaport, and is copied in Figure 1.


Figure 4.14 Knife-edge diffraction gain as a function of Fresnel diffraction parameter v .
Figure 1: Knife-edge diffraction gain in dB.
Expressions exist for the multiple knife edge diffraction problem - when multiple obstructions block a propagating wave. However, these are largely computed via numerical analysis, so we won't elaborate on them.

## 2 Rough Surface Scattering

When we discussed reflection, we said a wave was impinging on a flat surface. But most surfaces (e.g., walls, ground) are rough, not flat. When is a surface considered rough? When the maximum height "protuberance" from the surface, $h$, is greater than $h_{c}$,

$$
\begin{equation*}
h_{c}=\frac{\lambda}{8 \sin \theta_{i}} \tag{3}
\end{equation*}
$$

where $\theta_{i}$ is, again, the angle of incidence.
Scattering has two effects important to us:

1. Rough surface scattering reduces the power in the reflected wave.
2. Scattering causes additional multipath to be received in directions other than the specular direction (recall $\theta_{r}=\theta_{i}$ ).

For 1., if the surface is rough, then the reflected wave has reflection coefficient multiplied by $\rho_{s}$, so that $\Gamma_{\text {rough }}=\rho_{S} \Gamma$. Multiple
expressions exist to compute $\rho_{S}$. Two given in the book are:

$$
\begin{aligned}
\rho_{S} & =\exp \left[-8\left(\frac{\pi \sigma_{h} \sin \theta_{i}}{\lambda}\right)^{2}\right] \\
\rho_{S} & =\exp \left[-8\left(\frac{\pi \sigma_{h} \sin \theta_{i}}{\lambda}\right)^{2}\right] I_{0}\left[8\left(\frac{\pi \sigma_{h} \sin \theta_{i}}{\lambda}\right)^{2}\right]
\end{aligned}
$$

where $\sigma_{h}$ is the standard deviation of the height of the rough surface. The second expression is considered to be more accurate.

For 2., scattering is a very useful and impactful phenomena. Scattering is the basis of radar, weather systems, and passive RFID tags. Engineers have put much effort into understanding the phenomena of EM wave scattering.

Similar to Huygens' principal, the wave field at the scatterer is assumed to become a secondary re-radiator. However, the object (the scatterer) is assumed to absorb the incident power, and then re-radiate it (The re-radiation is not assumed to be occurring from the free space near the object). The object is considered a new point source, where power is "received" and "retransmitted". Essentially we have two links, one from the transmitter to the scatterer, and one away from the scatterer to the end receiver.

Typical airplane, weather radar is monostatic, i.e., the transmitter and receiver are co-located. But in wireless comm systems (and more robust airplane radar systems) the TX and RX are not in the same place. The bistatic radar equation describes the received power $P_{r}$ in a scattered wave at a receiver that is not at the transmitter location. In linear and dB terms,

$$
\begin{aligned}
P_{r}= & \frac{P_{t} G_{t} G_{r} \sigma_{R C S} \lambda^{2}}{(4 \pi)^{3} d_{t}^{2} d_{r}^{2}} \\
P_{r}(\mathrm{dBW})= & P_{t}(\mathrm{dBW})+G_{t}(\mathrm{~dB})+G_{r}(\mathrm{~dB})+\sigma_{R C S}\left(\mathrm{dBm}^{2}\right) \\
& +20 \log _{10} \lambda^{2}-30 \log _{10} 4 \pi-20 \log _{10} d_{t}-20 \log _{10} d_{r}
\end{aligned}
$$

Note that there are two $1 / d^{2}$ terms, one corresponding to each "link" described above. The $\sigma_{R C S}$ term has units of $m^{2}$ and is the radar cross section of the scatterer. It is an area, like an antenna effective aperture, that describes how much power is absorbed by the scatterer to be reradiated. Also, note that in the dB expression, the $\mathrm{dBm}^{2}$ units cancel because there are two $\mathrm{dBm}^{2}$ terms on top $\left(\sigma_{R C S}\right.$ and $\left.20 \log _{10} \lambda^{2}\right)$ and two $\mathrm{dBm}^{2}$ terms on bottom $\left(20 \log _{10} d_{t}\right.$ and $\left.20 \log _{10} d_{r}\right)$.

## References

[1] W. L. Stutzman and G. A. Theile. Antenna Theory and Design. John Wiley \& Sons, 1981.

