

Lecture 9

Today: (1) Multipath Fading

- Exam 1: Mean=88.7, Median=91, $\sigma = 9.4$.
- Today's viewing: MUSE video; Today's reading: 5.1, 5.2, 5.4, 5.5.1.
For Tue: Rap 5.4.3, 5.6, 5.7, 5.8.

1 Multipath Fading

We've talked about physics, that is, how wave propagation and its interaction with the environment causes reflection, transmission, diffraction, and scattering. Many individual propagating waves arrive at the receiver, these waves are called *multipath components*, or collectively, *multipath*. These multipath cause *fading* effects (changes in the received power) grouped together and called *multipath fading*. There are many kinds of multipath fading.

The challenges caused by multipath fading in wireless communication systems are one the most significant challenges addressed by wireless engineers today. Engineers have developed a variety of modulation and diversity schemes in order to counteract the negative influence of multipath fading. And, we are developing methods which take advantage of multipath in particular ways as a benefit for communication systems. In my lab, we are using radios as multipath channel sensors which use fading to improve the security and self-awareness of wireless networks. All this to say, *understanding of the fundamentals of fading, its basic causes, the statistics of fading, and its temporal, spatial, and frequency-domain characteristics is a critical skill for wireless engineers.*

We're first going to talk about received power when multiple multipath signals are received at the receiver. Then, we'll present the spatial and temporal characteristics of multipath.

1.1 Multipath

We've been talking about the EM field. The E-field E_b when we send a sinusoid at frequency f_c through the channel is,

$$E_b = E_0 \cos(2\pi f_c t + \theta)$$

where E_0 is positive real number. In other words,

$$E_b = \Re \left[E_0 e^{j2\pi f_c t + j\theta} \right] = \Re \left[e^{j2\pi f_c t} E_0 e^{j\theta} \right]$$

The above expressions are called bandpass representation. When we want to write a simpler expression, we write the complex *baseband-equivalent representation*:

$$E = E_0 e^{j\theta}$$

and we can “translate” any complex baseband-equivalent signal into its bandpass signal by applying

$$E_b = \Re \left[e^{j2\pi f_c t} E \right]$$

The simpler expression E has more easily viewable magnitude (amplitude) E_0 and phase (angle) θ .

The voltage received by the antenna is proportional to E and has the same complex baseband or real-valued baseband representation. For example, $V = \alpha E V_0 e^{j\theta}$, for some constant α .

As we discussed, many such multipath wave components arrive at the receiver. *They add together as voltages*. DO NOT add the powers of the multipath together – there is no such physical antenna that add together the powers of multipath. If you find a way, patent it – quick – as we will see, this phasor (another word for complex) sum of voltages creates major problems for wireless communication systems.

Let’s say there are M multipath components, numbered 0 through $M - 1$. Component i has amplitude V_i and phase θ_i . Then the total voltage at the receiver antenna will be:

$$V_{TOT} = \sum_{i=0}^{M-1} V_i e^{j\theta_i}$$

The received power is then proportional to $|V_{TOT}|^2$ (by a resistance).

1.2 Temporal

Let’s expand on where these phase angles come from. Recall that $V_i e^{j\theta_i}$ is the representation of $V_i \cos(2\pi f_c t + \theta_i)$. If $V_i \cos(2\pi f_c t)$ is transmitted from the transmitter antenna, how do the phases of the multipath components behave with respect to each other? Well, each component has its own path length. It really did travel that length. And EM waves all travel at the same speed (in air) – $c = 3 \times 10^8$ m/s. So some waves arrive later than others. Let τ_i denote the time delay of arrival for multipath i relative to the transmit time. It is d_i/c , where d_i is the length of component i . What happens when a function is delayed by τ_i ? We replace t with $t - \tau_i$ in the function. So $V_i \cos(2\pi f_c(t - \tau_i))$ is received. Well, not the full story – reflections and diffractions also cause phase changes (we discussed specifics for reflection in Section 4.5). Really,

$$V_i \cos(2\pi f_c(t - \tau_i) + \phi_i)$$

is received, where ϕ_i is the sum of all phase changes caused by the physical propagation phenomena. The above equation is in baseband notation, what is the complex baseband notation? It is:

$$V_i e^{j(-2\pi f_c \tau_i + \phi_i)}.$$

So what is the total received voltage from all multipath?

$$V_{TOT} = \sum_{i=0}^{M-1} V_i e^{j(-2\pi f_c \tau_i + \phi_i)} \quad (1)$$

In other words, $\theta_i = -2\pi f_c \tau_i + \phi_i$. We've now written it in terms of its temporal delay, τ_i . Note that V_{TOT} has incidentally become a function of frequency f_c .

1.3 Channel Impulse Response

What we have in (1) is a frequency response as a function of frequency f_c . The equation can show the frequency response at *any* frequency. Let $f = f_c$, maybe that makes it clearer. So (1) is a frequency-domain representation of the total voltage. Let's convert to the time domain. How? Using the inverse Fourier transform:

$$\mathfrak{F}^{-1} \left\{ \sum_{i=0}^{M-1} V_i e^{j(-2\pi f \tau_i + \phi_i)} \right\} = \sum_{i=0}^{M-1} V_i \mathfrak{F}^{-1} \left\{ e^{j(-2\pi f \tau_i + \phi_i)} \right\} = \sum_{i=0}^{M-1} V_i e^{j\phi_i} \delta(\tau - \tau_i)$$

What this says is that in the *time delay domain*, the arrivals of multipath i occurs at delay τ_i .

This leads to how we frame the channel: as an echo-causing filter, with an impulse response that is a sum of time-delayed impulses. Let $s(t)$ be the transmitted signal and $r(t)$ be the received signal. Then

$$r(t) = \frac{1}{2} s(t) \star h(\tau)$$

where $h(\tau)$ is called the *channel impulse response* (CIR), and is given by

$$h(\tau) = \sum_{i=0}^{M-1} a_i e^{j\phi_i} \delta(\tau - \tau_i) \quad (2)$$

The a_i are proportional to V_i but are unitless – the units are contained in $s(t)$, which has units of Volts. The amplitude $|a_i|$ is the amplitude gain in that path; the squared magnitude $|a_i|^2$ is the power gain in that path. Researchers often plot the squared magnitude of $h(\tau)$ in the dB domain and call it the *power delay profile*, or PDP. We also denote this power in the i th path as $P(\tau_i) = |a_i|^2$.

We measured many of these in my lab. For example, Figure 1 shows three examples.

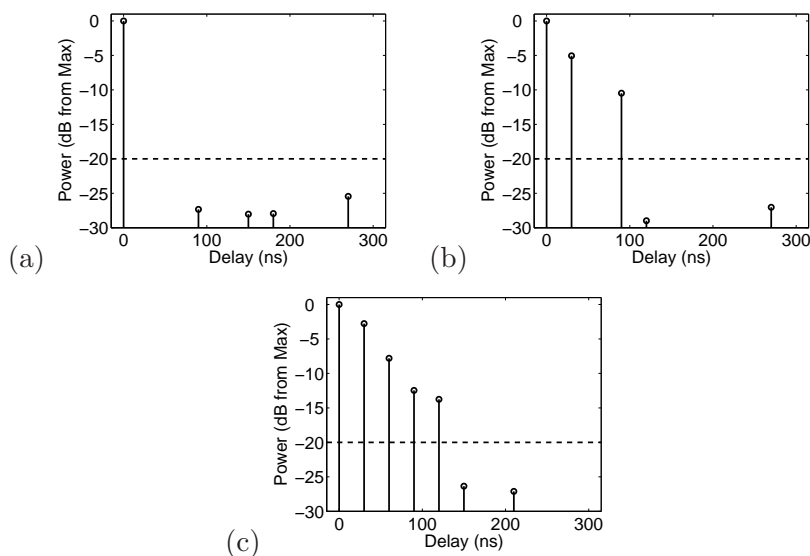


Figure 1: Measured power delay profiles (power gain normalized to the power gain of the maximum power path) in Salt Lake City (a) in residential area W of U. campus; (b) 4th S commercial district; (c) Main St., downtown [1].

1.4 Received Power

What happens to the transmitted signal $s(t)$ when it is convolved with $h(\tau)$? Well, many copies show up, at different time delays:

$$\tilde{s}(t) = \sum_{i=0}^{M-1} a_i e^{j\phi_i} s(t - \tau_i)$$

For example, what if a rectangular pulse was sent (in Digital Communications, many symbols look like a pulse)? Let $s(t) = \text{rect}\left[\frac{t}{T_b} - \frac{1}{2}\right]$. In this case,

$$\tilde{s}(t) = \sum_{i=0}^{M-1} a_i e^{j\phi_i} \text{rect}\left[\frac{t - \tau_i}{T_s} - \frac{1}{2}\right]$$

where T_s is the symbol duration. Essentially, we have versions of the rect pulse piling on top of each other.

CASE 1: $\tau_i \ll T_s$: If the τ_i s are small compared to T_s ,

$$\tilde{s}(t) \approx \left(\sum_{i=0}^{M-1} a_i e^{j\phi_i} \right) \text{rect}\left[\frac{t}{T_s} - \frac{1}{2}\right]$$

then we have all of the pulses adding together, in a phasor sum. The sum might be constructive or destructive. But it acts on the whole pulse.

CASE 2: τ_i aren't small compared to T_s : We will have intersymbol interference, and will need an equalizer in our receiver.

Note T_s is designed by us (the engineer). Why not make T_s big? Because the symbol rate is $1/T_s$! You slow down data rate when increasing T_s .

1.5 Time Dispersion Parameters

There are lots of τ_i so lets provide a number with more specificity, a single number that summarizes the size of the time dispersion: the RMS delay spread, σ_τ ,

$$\begin{aligned}\sigma_\tau &= \sqrt{\bar{\tau}^2 - \bar{\tau}^2} \\ \bar{\tau} &= \frac{\sum_i |a_i|^2 \tau_i}{\sum_i |a_i|^2} = \frac{\sum_i P(\tau_i) \tau_i}{\sum_i P(\tau_i)} \\ \bar{\tau}^2 &= \frac{\sum_i |a_i|^2 \tau_i^2}{\sum_i |a_i|^2} = \frac{\sum_i P(\tau_i) \tau_i^2}{\sum_i P(\tau_i)}\end{aligned}$$

One good rule of thumb is that the receiver doesn't need an equalizer if $\sigma_\tau \leq 0.1T_s$.

References

- [1] D. Maas, M. H. Firooz, J. Zhang, N. Patwari, and S. K. Kasera. Channel sounding for the masses: Low complexity GNU 802.11b channel impulse response estimation. *IEEE Trans. Wireless Communications*, 11(1):1–8, Jan. 2012.