ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 10

Today: (1) Fade Distribution, (2) Fading Demo

- Today: Molisch 5.2, 5.3. Thu: Molisch 5.4-5.6.
- HW 4 due today at noon. Homework 5 due Feb 21 at 9:10 am.

1 Fade Distribution

This section describes provides quantitative ways to design reliable wireless links in the presence of multipath fading. To proceed with this quantification, we need probabilistic analysis, which requires us to move from considering purely "specular" multipath to combinations of specular and "diffuse" multipath:

- *Specular multipath*: What we've been talking about: individual multipath components, of which we said there were M total, each with its own amplitude and phase.
- *Diffuse multipath*: multipath which are each very low in power, but there are too many to consider individually. In mathematical analysis, we say they have infinitely small power, but there are infinitely many of them. Typically used to simulate the many many multipath that do arrive due to scattering and diffraction. It is easier to analyze fading when we can lump lots of multipath into this "diffuse" camp, rather than specifying each individual multipath and how much power each one has.

1.1 Rayleigh Fading

Main idea: all multipath are diffuse. These diffuse multipath have small amplitude and random phase. Using the central limit theorem, and some probability (ECE 5510), we can come up with a pdf of the magnitude of the received voltage, $r = |V_{TOT}|$.

Here, the name of the random variable is R and the value r represents some number of interest.

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), & r \geq 0 \\ 0, & o.w. \end{cases} \quad (1)$$

where σ is a parameter that can be set as needed. For example, if you know the mean value of the power, that is, $E\left[R^2\right]$ is $\bar{r^2}$, then you would set $2\sigma^2 = \bar{r^2}$.

The most important function for the communication system designer is the cumulative distribution function (CDF). It tells us the probability that the envelope will fall below some value r.

$$F_R(r) = P[R \leq r] = \begin{cases} 1 - \exp\left(-\frac{r^2}{2\sigma^2}\right), & r \geq 0 \\ 0, & o.w. \end{cases} \quad (2)$$

Often you want to know the probability that R falls X dB below the average power. Since the average power is $2\sigma^2$, this means $10\log_{10}(2\sigma^2) - X$ dB or $2\sigma^2 10^{-X/10}$ in linear terms.

<u>**Def'n:**</u> *Fade Margin*

the additional power loss which is included in the link budget in order to keep the received signal power higher than the minimum, with high probability.

Example: Fade margin design: Rayleigh case

Assume that it is acceptable to have the received power fall below the receiver sensitivity 1% of the time, and that the channel is Rayleigh. What fade margin (in dB) is required?

Solution: We are given $P[R \leq r] = 0.01$. We want to find the threshold in terms of the mean received signal level, so setting $r_{mean} = 1$, we have that $\sigma = 0.7979$. Then, solving (2) for r,

$$\begin{aligned} \exp\left(-\frac{r^2}{2\sigma^2}\right) &= 0.99 \\ r &= \sqrt{-2\sigma^2 \ln 0.99} = 0.1131 \end{aligned} \quad (3)$$

If the envelope falls to 0.1131, then the power falls to $(0.1131)^2 = 0.0128$, which is equal to -18.9 dB. Since the mean value is 1, or 0 dB, the required fade margin is $0 - (-18.9) = 18.9$ dB.

1.2 Ricean fading

Main idea: One specular path with amplitude $V_0 = A$ plus diffuse power.

The Rice distribution is named after its creator, Stephen O. Rice. I have his 1945 paper in my office if you want to use it for a project. Note that people just can't agree on the spelling of the adjective made out of his name, whether it is "Rician" or "Ricean". Google gives me 249k results for "Rician" and 217k results for "Ricean". Why not pick one? Who knows.

The pdf of Ricean fading is

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{r^2+A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right), & r \geq 0 \\ 0, & o.w. \end{cases} \quad (4)$$

where A is the peak amplitude of the specular (dominant) signal, and $I_0(\cdot)$ is the modified Bessel function of the first kind and zeroth order. Note Matlab has a `besseli(0, x)` function. The Ricean K-factor is defined as the ratio between the power in the specular components to the power in the diffuse components,

$$K = \frac{A^2}{2\sigma^2}, \quad \text{or } K(\text{dB}) = 10 \log_{10} \frac{A^2}{2\sigma^2}. \tag{5}$$

Note that for $K = 0$ or $K(dB) = -\infty$, the Ricean pdf is the same as the Rayleigh pdf.

There isn't an analytical expression for the CDF, analogous to (2), unfortunately. So we need to use a table or a figure. The Molisch book only has a CDF plot for $K = 0.3$, 3, and 20 dB. I am including in Figure 1 a more complete plot. Keep this figure with your notes, it will be included in Exam 2. It also includes Rayleigh, because Rayleigh is the same as Ricean when $K = -\infty$.

Example: Fade margin design: Ricean case
Assume that it is acceptable to have the received power fall below the receiver sensitivity 1% of the time, and that the channel is Ricean, with K-factor 3 dB, 9 dB, or 15 dB. What fade margin (in dB) is required?
Solution: From the chart, on the horizontal line from 1%, the fade margin would be -15.5 dB, -7.0 dB, and -3 dB, for K-factor of 3 dB, 9 dB, and 15 dB, respectively.

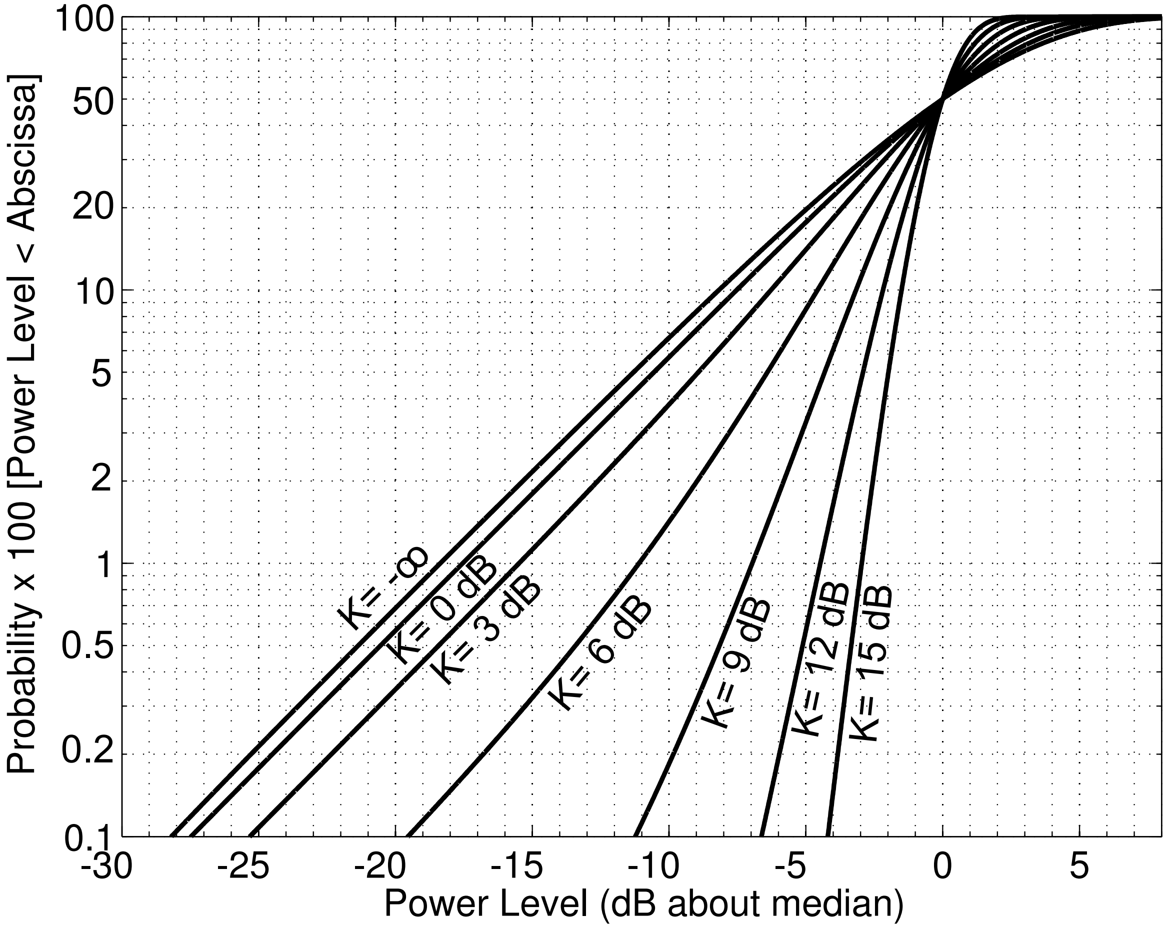

Figure 1: Ricean CDF for various K-factors. Note "Power Level" is $20 \log_{10} \frac{r}{r_{median}}$, and the y-axis is the probability $\times 100$ (percentage) that the power level (fading gain compared to the median) is less than the value on the abscissa. Note the mean is about 1 dB higher than the median.