Lecture 10

Today: (1) Fade Distribution, (2) Doppler

• OH Thu 10-noon, Fri 9-noon (J. Kemp)
• Today’s reading: Rap 5.4.3, 5.6, 5.7. Nix 5.8, except for a project. Thu reading: start into Chapter 6.

1 Fade Distribution

Specular multipath: What we’ve been talking about: individual multipath components, of which we said there were \( M \) total, each with its own amplitude and phase.

Diffuse multipath: multipath which are each infinitely low in power, but there are infinitely many of them. Typically used to simulate the many many multipath that do arrive due to scattering and diffraction. It is easier to talk about fading when we can lump lots and lots of multipath into this “diffuse” camp, rather than specifying each individual multipath and how much power each one has.

1.1 Rayleigh Fading

Main idea: all multipath are diffuse. These diffuse multipath have small amplitude and random phase. Using the central limit theorem, and some probability (ECE 5510), we can come up with a pdf of the magnitude of the received voltage, \( r = |V_{TOT}| \).

Here, the name of the random variable is \( R \) and the value \( r \) represents some number of interest.

\[
f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp \left( -\frac{r^2}{2\sigma^2} \right), & r \geq 0 \\ 0, & \text{o.w.} \end{cases} \tag{1}
\]

where \( \sigma \) is a parameter that can be set as needed. For example, if you know the mean value of the signal magnitude is \( r_{\text{mean}} \), then you would set \( \sigma = \sqrt{\frac{2}{\pi}} r_{\text{mean}} \).

The most important function for the communication system designer is the cumulative distribution function (CDF). It tells us the probability that the envelope will fall below some value \( r \).

\[
F_R(r) = P[R \leq r] = \begin{cases} 1 - \exp \left( -\frac{r^2}{2\sigma^2} \right), & r \geq 0 \\ 0, & \text{o.w.} \end{cases} \tag{2}
\]
**Def’n:** Fade Margin  
the additional power loss which is included in the link budget in order to keep the received signal power higher than the minimum, with high probability.

**Example: Fade margin design: Rayleigh case**  
Assume that it is acceptable to have the received power fall below the receiver sensitivity 1% of the time, and that the channel is Rayleigh. What fade margin (in dB) is required?  
**Solution:** We are given $P[R≤r]=0.01$. We want to find the threshold in terms of the mean received signal level, so setting $r_{mean}=1$, we have that $\sigma=0.7979$. Then, solving (2) for $r$,

$$\exp\left(-\frac{r^2}{2\sigma^2}\right) = 0.99$$

$$r = \sqrt{-2\sigma^2 \ln 0.99} = 0.1131$$

(3)

If the envelope falls to 0.1131, then the power falls to $(0.1131)^2 = 0.0128$, which is equal to $-18.9$ dB. Since the mean value is 1, or 0 dB, the required fade margin is $0 - (-18.9) = 18.9$ dB.

1.2 Ricean fading

Main idea: One specular path with amplitude $V_0 = A$ plus diffuse power.

The Rice distribution is named after its creator, Stephen O. Rice. I have his 1945 paper in my office if you want to use it for a project. Note that people just can’t agree on the spelling of the adjective made out of his name, whether it is “Rician” or “Ricean”. Google gives me 104k results for “Rician” and 80k results for “Ricean”. I think that the mathematicians use “Ricean” more while the radio engineers use “Rician” more. Why? Who knows.

The pdf of Ricean fading is

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2+A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right), & r \geq 0 \\ 0, & \text{o.w.} \end{cases}$$

(4)

where $A$ is the peak amplitude of the specular (dominant) signal, and $I_0(.)$ is the modified Bessel function of the first kind and zeroth order. Note Matlab has a `besseli(0,x)` function. The Ricean $K$-factor is defined as the ratio between the power in the specular components to the power in the diffuse components,

$$K = \frac{A^2}{2\sigma^2}, \quad \text{or} \quad K(\text{dB}) = 10 \log_{10} \frac{A^2}{2\sigma^2}.$$  

(5)
Note that for $K = 0$ or $K(dB) = -\infty$, the Ricean pdf is the same as the Rayleigh pdf.

There isn’t an analytical expression for the CDF, analagous to (2), unfortunately. So we need to use a table or a figure. The Rappaport book only has a CDF plot for $K = 6$ dB. I am including in Figure 1 a more complete Figure. Keep this figure with your notes, it will be included in Exam 2. It also include Rayleigh as the $K = -\infty$ line.

Figure 1: Ricean CDF for various $K$-factors. Note “Power Level” is $20 \log_{10} \frac{r}{r_{median}}$, and the y-axis is the probability $\times 100$ (percentage) that the power level (fading gain compared to the median) is less than the value on the abscissa.

Example: Fade margin design: Ricean case
Assume that it is acceptable to have the received power fall below the receiver sensitivity 1% of the time, and that the channel is Ricean, with $K$-factor 3, 9, or 15. What fade margin (in dB) is required?
2 Doppler Fading

So far we’ve talked about fading without movement. A static link has a fading loss. But for a link in motion, (1) the fading loss changes over time, and (2) the frequency shifts. Why is this?

In lecture 9, we came up with the top expression for the complex baseband received voltage:

\[ V_{TOT} = M - 1 \sum_{i=0}^{M-1} V_i e^{j(-2\pi f_c \tau_i + \phi_i)} \]

The second expression is rewritten with \( d_i = c \tau_i \), where \( d_i \) is the distance multipath component \( i \) travels. (Is this the straight-line from the TX to RX? Or the length of the line following the path? Answer: the latter.) This showed us the frequency dependence of the fading channel gain, which is \( 20 \log_{10} |V_{TOT}| \). Now, let’s talk about what happens when the receiver is moving. Motion causes the time delays to change, because the distance that the wave must travel is becoming either shorter or longer. How much shorter or longer?

Let the angle of arrival of component \( i \) be \( \theta_i \). (Consider several multipath components and their geometry.) Let’s assume I move \( \Delta_{move} \) meters in the direction \( \theta_{move} \). We assume that these waves are effectively plane waves in the local (small) area near the antenna. Thus the only thing that changes when I move the antenna to a new position is that the wave is shortened or lengthed by a factor of the distance I moved, multiplied by the cosine of the angle in between \( \theta_i \) and my direction of motion \( \theta_{move} \). After my movement of \( \Delta_{move} \) meters in the direction \( \theta_{move} \), my \( V_{TOT} \) becomes:

\[ V_{TOT} = \sum_{i=0}^{M-1} V_i \exp \left\{ j \left( -\frac{2\pi}{\lambda_c} [d_i + \Delta_{move} \cos(\theta_{move} - \theta_i)] + \phi_i \right) \right\} \]

2.1 One Component

First, keep things simple by considering only one arriving multipath.

\[ V_{TOT} = V_0 \exp \left\{ j \left( -\frac{2\pi}{\lambda_c} [d_0 + \Delta_{move} \cos(\theta_{move} - \theta_0)] + \phi_0 \right) \right\} \]

Let’s consider when the antenna is moving at a constant velocity \( v \) meters per second. Then, at time \( t \), we have \( \Delta_{move} = vt \). So,

\[ V_{TOT} = V_0 \exp \left\{ j \left( -\frac{2\pi}{\lambda_c} [d_0 + vt \cos(\theta_{move} - \theta_0)] + \phi_0 \right) \right\} \]
There’s a lot going on here, but we actually have not just a phase, but a complex sinusoid of,

\[
f_d = -\frac{v \cos(\theta_{\text{move}} - \theta_0)}{\lambda_c}
\]  

(6)

This is called the Doppler shift. When one sends a frequency \(f_c\), the received frequency of the signal has shifted up (or down, if \(f_d\) is negative) by this factor.

1. What happens when I move such that \(\theta_i = \theta_{\text{move}}\)?

2. What happens when I move such that \(\theta_i = \theta_{\text{move}} + 90\) degrees?

3. What happens when I move such that \(\theta_i = \theta_{\text{move}} + 180\) degrees?

The worst case is called the maximum Doppler frequency, \(f_m = |f_d| = v/\lambda_c\).

**Example:** What are the maximum Doppler frequencies for a mobile in a vehicle on I-15, at 850 and 1950 MHz?

**Solution:** Given 1 mile per hour = 0.447 meters per second, let’s say a maximum speed of 80 miles/hr, which gives \(v = 35.8\) m/s. Then \(f_m = 101\) Hz, or 232 Hz, for 850 or 1950 MHz, respectively.

### 2.2 Many Components

Now, each component contributes a complex sinusoid of frequency \(f_d = -\frac{v \cos(\theta_{\text{move}} - \theta_0)}{\lambda_c}\) to the sum \(V_{TOT}\). The effect is frequency spreading. That is, for each frequency that is sent, many frequencies are received. In the frequency domain, the frequency content of the signal (say, \(S(\omega)\)) is convolved with the Doppler spectrum \(V_{TOT}(\omega)\).

If there are diffuse multipath coming equally from all directions, then the power spectrum of \(V_{TOT}\) can be determined to be:

\[
S(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left(\frac{f - f_c}{f_m}\right)^2}}
\]

which is shown in Figure 5.20 on page 219 in Rappaport.

### 2.3 System Design Implications

The Doppler spread is said to have an effect when the symbol duration is long compared to \(1/f_m\). Specifically, Rappaport recommends \(T_C = 0.423 f_m\), and if \(T_s > T_C\), that is, the symbol rate is higher than then there you have “fast fading”, and demodulation will be
difficult. Instead, one should design systems with $T_s << T_C$. Typically, this is not a problem.

See Figure 5.15 on page 211 in Rappaport for an example of Rayleigh fading. How often does the power level drop cross a horizontal “threshold” line? This is the level crossing rate (LCR). For the Clarke AOA spectrum model, the average number of level crossings per second is

$$N_R = \sqrt{2\pi f_m \rho e^{-\rho^2}}$$

Another useful statistic is how long the signal stays below this threshold when it goes below it. This is the average fade duration, denoted $\bar{t}_{fade}$ (or $\bar{\tau}$ in Rappaport). Again, for the Clarke AOA spectrum model,

$$\bar{t}_{fade} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

**Example: Fading statistics**

Let $1/T_s = 1 MHz$, and use the solution for 1950 MHz highway driving above. Find whether we have fast or slow fading. What is the fade duration, LCR, and $\bar{t}_{fade}$?