ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 11

Today: (1) Doppler, (2) Diversity

- HW 5 due one week from today (THURSDAY). It is for three lectures of HW questions. Discuss deadline time?
- Today's lecture is the last new topic for Exam 2, which is Feb 26.
- Reading: Today – Mol 13.1, 13.2, 13.4. Thu: Mol 11, up to 11.3.6

1 Doppler Fading

As we've mentioned, when a transmitter or receiver (or both) moves, the phases of the multipath components also changes, each at a different rate. Recall that

$$
\begin{aligned}
V_{TOT} &= \sum_{i=0}^{M-1} V_i e^{j(-2\pi f_c \tau_i + \phi_i)} \\
V_{TOT} &= \sum_{i=0}^{M-1} V_i e^{j(-2\pi d_i/\lambda_c + \phi_i)}
\end{aligned}
$$

The second expression is rewritten with $d_i = c\tau_i$, where d_i is the distance multipath component i travels. (Is this the straight-line from the TX to RX? Or the length of the line following the path? Answer: the latter.) This showed us the frequency dependence of the fading channel gain, which is $20\log_{10}|V_{TOT}|$. Now, let's talk about what happens when the receiver is moving. Motion causes the time delays to change, because the distance that the wave must travel is becoming either shorter or longer. How much shorter or longer?

Let the angle of arrival of component i be θ_i, like the one shown in Figure 1. (Actually, consider that multipath components will arrive with different θ_i.) Let's assume I move Δ_{move} meters in the direction θ_{move}. We assume that these waves are effectively plane waves in the local (small) area near the antenna. Thus the only thing that changes when I move the antenna to a new position is that the wave is lengthened (or shortened) by a factor of the distance I moved, multiplied by the cosine of the angle in between θ_i and my direction of motion θ_{move}. After my movement of Δ_{move} meters in the direction θ_{move}, my V_{TOT} becomes:

$$
V_{TOT} = \sum_{i=0}^{M-1} V_i \exp\left\{ j\left(-\frac{2\pi}{\lambda_c}[d_i + \Delta_{move}\cos(\theta_{move} - \theta_i)] + \phi_i \right) \right\}
$$

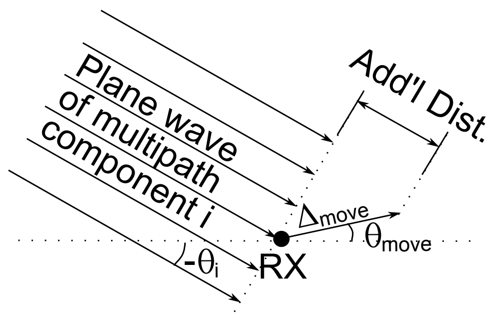

Figure 1: The plane wave of a multipath component arrives at the receiver (•). Based on the difference between the angle of movement θ_{move} and the angle of arrival of the multipath component θ_i, the multipath component distance di increases by $\cos(\theta_{move} - \theta_i)$ multiplied by the distance of travel of the receiver.

1.1 One Component

First, keep things simple by considering only one arriving multipath.

$$V_{TOT} = V_0 \exp\left\{ j\left(-\frac{2\pi}{\lambda_c}[d_0 + \Delta_{move}\cos(\theta_{move} - \theta_0)] + \phi_0 \right) \right\}$$

Let's consider when the antenna is moving at a constant velocity v meters per second. Then, at time t, we have $\Delta_{move} = vt$. So,

$$V_{TOT} = V_0 \exp\left\{ j\left(-\frac{2\pi}{\lambda_c}[d_0 + vt\cos(\theta_{move} - \theta_0)] + \phi_0 \right) \right\}$$

There's a lot going on here, but we actually have not just a phase, but a complex sinusoid of,

$$f_d = -\frac{v\cos(\theta_{move} - \theta_0)}{\lambda_c} \quad (1)$$

This is called the *Doppler shift*. When one sends a frequency f_c, the received frequency of the signal has shifted up (or down, if f_d is negative) by this factor.

1. What happens when I move such that $\theta_i = \theta_{move}$?
2. What happens when I move such that $\theta_i = \theta_{move} + 90$ degrees?
3. What happens when I move such that $\theta_i = \theta_{move} + 180$ degrees?

The worst case is called the maximum Doppler frequency, $f_m = |f_d| = v/\lambda_c$.

Example: What are the maximum Doppler frequencies for a mobile in a vehicle on I-15, at 850 and 1950 MHz?

1.2 Many Components

Now, each component contributes a complex sinusoid of frequency $f_d = -\frac{v\cos(\theta_{move}-\theta_i)}{\lambda_c}$ to the sum V_{TOT}. The effect is *frequency spreading*. That is, for each frequency f_c that is sent, many frequencies are received in the range $f_c - f_m$ to $f_c + f_m$. In the frequency domain, the frequency content of the signal (say, $S(\omega)$) is convolved with the Doppler spectrum $V_{TOT}(\omega)$). This may cause two signals that were originally non-overlapping in frequency to now overlap in frequency.

1.3 System Design Implications

The Doppler spread is said to have an effect when the symbol duration is long compared to $1/f_m$. Specifically, let the coherence time T_C be defined as $T_C = 0.423\frac{1}{f_m}$. If $T_s > T_C$, then you have "fast fading", and demodulation will be difficult, because the channel changes during one symbol period. Instead, one should design systems with $T_s << T_C$, (for "slow fading") or equivalently, a symbol rate $1/T_s$ that is much greater than $1/T_C$. Typically, this is not a problem, except for some OFDM systems which have long T_s.

See Figure 5.15 on page 211 in Rappaport for an example of Rayleigh fading. How often does the power level drop cross a horizontal "threshold" line? This is the *level crossing rate* (LCR). For the Clarke AOA spectrum model, the average number of level crossings per second is

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2},$$

where $\rho = r/R_{rms}$, and R_{rms} is the root-mean square (RMS) average signal amplitude, and r is the amplitude "level" you are using. For example, to check crossings that are 10 dB down from the RMS received amplitude, use $\rho = 10^{-10/10} = 0.1$. Or, to check crossings that are 30 dB down from the RMS received *power*, use $\rho = 10^{-30/20} = 0.032$. Another useful statistic is how long the signal stays below this threshold when it goes below it. This is the *average fade duration*, denoted $\bar{t}_{fade}$ (or $\bar{\tau}$ in Rappaport). Again, for the Clarke AOA spectrum model,

$$\bar{t}_{fade} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

Example: Fading statistics
Let $1/T_s = 1MHz$, and use the solution for 1950 MHz highway driving above. What is the average fade duration and level crossing rate for a fade that is 20 dB lower in power than the RMS received power?

2 Antenna Diversity

When we send a signal in a multipath fading channel, we are taking a chance and hoping that the fading is not more severe than our fade margin. The big picture idea of diversity is "don't put all of your eggs in one basket". Diversity is the use of multiple channels to increase the signal to noise ratio in the presence of random fading losses. Antenna diversity is the use of multiple antennas on a transceiver in order to create these multiple channels.

For fading channels, we know that there is a finite probability that a signal power will fall below any given fade margin. For a Rayleigh channel, we showed that to have the signal above the required SNR 99% of the time, we needed to include a fade margin of 18.9 dB. This is a big "loss" in our link budget. For example, if we didn't need to include the fade margin in the link budget, we could multiply the path length by a factor of $10^{18.9/20} \approx 10$ (in free space); or increase the number of bits per symbol in the modulation significantly higher so that we can achieve higher bit rate for the same bandwidth.

There are several physical means to achieve multiple channels, and to get the received power on those channels to be nearly independent. Each has its advantages and disadvantages. In this section we study methods using multiple antennas.

After this lecture, you should have three critical skills:

1. Understand what is meant by space, polarization, and angle diversity, and the benefits and drawbacks of each method in a wireless communications system.
2. Understand how to combine the signals from multiple channels, including scanning, selection, equal gain, and maximal ratio combining methods.
3. Know the effect on system design: Be able to calculate the probability of outage or required fade margin when using a particular diversity combining scheme, assuming Rayleigh fading.

2.1 Methods for Antenna Diversity

2.1.1 Space Diversity

Space diversity at a receiver is the use of multiple antennas shifted in space. Because multipath fading changes quickly over space the signal amplitude on the antennas can have a low correlation. The low correlation typically comes at separation distances of more than half the wavelength. The Jakes model (equal power from all angles) says that the correlation coefficient at $\lambda/2$ is exactly zero; however, in reality, this is not true. The actual angular power profile (multipath power vs. angle) determines the actual correlation coefficient. In general, we either accept that the correlation coefficient is not perfectly zero, or we separate the antennas further than $\lambda/2$. Or, add some gain pattern diversity, *i.e.*, using different gain patterns to help decorrelate the signals.

The problems with space diversity are most importantly that for consumer radios, we want them to be small; and multiple antennas shifted on the order of λ between them means that the device will be larger. This is fine when space is not a big concern – for base stations, or for laptops or even access points. Note that no additional signal needs to be transmitted. There is no limit on the number of antennas that can be used.

Space diversity can also be used at a transmitter. This is a topic we will cover when we discuss multiple-input multiple output (MIMO) communications systems, including showing how Alamouti coding can be used to use the multiple antennas at a transmitter to achieve space diversity.

2.1.2 Polarization and Angle Diversity

Polarization diversity is the use of two antennas with different polarizations. We know that reflection coefficients are different for horizontal and vertically polarized components of the signal. Scattered and diffracted signal amplitudes and phases also are different for opposite polarizations. Thus we can consider one polarized signal, which is the sum of the amplitudes and phases of many reflected, scattered, and diffracted signals, to be nearly uncorrelated with the other polarized signal.

Angle diversity is the use of multiple antennas with different antenna radiation patterns. See Molisch Figure 13.7. Each pattern will provide more gain to multipath arriving in its mainlobe, and less to those arriving from other directions. Different patterns will provide different gains to different multipath, and thus the V_{TOT} for the different antennas will be different.

The advantages of polarization and angle diversity is that the two antennas don't need to be spaced $\lambda/2$ apart (although, the two antennas must be designed together). These methods may be combined with space diversity so to further reduce the correlation coefficient between the signal received at two antennas. Polarization diversity, like space diversity, doesn't require any additional bandwidth or power from the transmitter.

The disadvantage of polarization diversity is that there can be only two channels – vertical and horizontal (or equivalently, right-hand and left-hand circular) polarizations. There is a more subtle limit on angle diversity schemes – for a finite beamwidth, we can only have a finite number of independent antenna patterns.

2.2 Diversity Combining

In the previous section, we described how we might achieve M different (nearly) independent channels. In this section, we discuss what to do with those independent signals once we get them. These are called combining methods. *We need them regardless of diversity method.* We only want one bitstream, so somehow we need to combine the channels' signals together. Here are some options, in order of complexity:

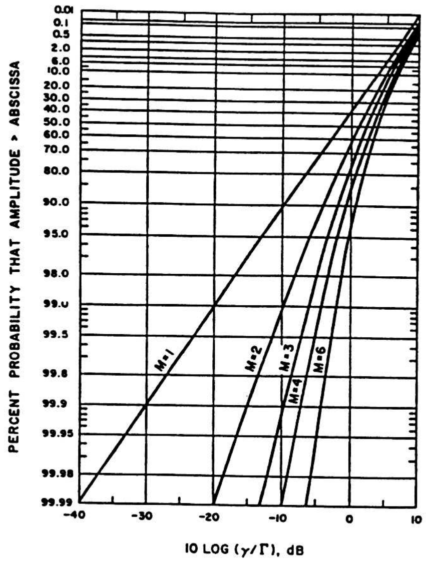

Figure 7.11 Graph of probability distributions of *SNR* = γ threshold for *M* branch selection diversity. The term Γ represents the mean *SNR* on each branch [from [Jak71] © IEEE].

Figure 2: Rappaport Figure 7.11, the impact of selection combining.

1. *Scanning / Switching Combiner*: Scan among the channels, changing when the current SNR goes below the threshold.
2. *Selection Combiner*: Select the maximum SNR channel's signal and use only it.
3. *Equal Gain Combiner*: Co-phase the signals and then add them together.
4. *Maximal Ratio Combiner*: The optimal solution in terms of SNR – co-phase and weight (multiply) each signal by the square root of its signal to noise ratio (SNR), and then add them together.

"Co-phase the signals" means that we need to multiply signals by $e^{j\phi_i}$ for some constant phase angle ϕ_i on channel i, so that the (otherwise random) phases of the signals on the different channels line up. If we don't co-phase the signals before combining them, we end up with the same multipath fading problem we've always had - signals sometimes add together destructively. You should be prepared to describe any of these combining methods, and discuss its effect on the fade margin required for a link.

2.2.1 Selection Combining

Let's say that we have M statistically independent channels. This independence means that one channel's fading does not influence, or is not correlated in any way with, another channel's fading.

Let's assume that each channel is Rayleigh with identical mean SNR Γ. At any given instant, the SNR on channel i is denoted γ_i. Based on the Rayleigh assumption for γ_i, it has a CDF of:

$$P[\gamma_i \leq \gamma] = 1 - e^{-\gamma/\Gamma}$$

This means that the probability that the SNR on channel i is less than the threshold γ is given by $1 - e^{-\gamma/\Gamma}$, where again, Γ is the mean SNR for the channel. In past lectures, we showed that we can determine a fade margin for a single channel ($M = 1$) based on this equation. For example, setting the probability of being less than the threshold to 1%,

$$\begin{aligned} 0.01 &= 1 - e^{-\gamma/\Gamma} \\ 0.99 &= e^{-\gamma/\Gamma} \\ \gamma &= \Gamma(-\ln 0.99) = \Gamma(0.0101) = \Gamma(\text{dB}) - 19.98(\text{dB}) \end{aligned} \tag{2}$$

Thus compared to the mean SNR on the link, we need an additional 20 dB of fade margin (this is slightly less when we use the median SNR).

In contrast, in selection combining, we only fail to achieve the threshold SNR when *all* channels are below the threshold SNR. Put in another way, if any of the channels achieve good enough SNR, we'll select that one, and then our SNR after the combiner will be good enough. What is the probability all of the M channels will fail to achieve the threshold SNR γ? All M channels have to have SNR below γ. The probability is the product of each one:

$$P[\gamma_i < \gamma, \forall i = 1, \ldots, M] = \left[1 - e^{-\gamma/\Gamma}\right] \cdots \left[1 - e^{-\gamma/\Gamma}\right] = \left[1 - e^{-\gamma/\Gamma}\right]^M$$

Example: What is the required fade margin when assuming Rayleigh fading and $M = 2$ independent channels, for a 99% probability of being above the receiver threshold?

Again, set 0.01 equal, this time, to $\left[1 - e^{-\gamma/\Gamma}\right]^2$, so

$$\begin{aligned} 0.1 &= 1 - e^{-\gamma/\Gamma} \\ 0.9 &= e^{-\gamma/\Gamma} \\ \gamma &= \Gamma(-\ln 0.9) = \Gamma(0.1054) = \Gamma(\text{dB}) - 9.77(\text{dB}) \end{aligned} \tag{3}$$

So the fade margin has gone down to less than 10 dB, a reduction in fade margin of 10 dB!

As M increases beyond 2, you will see diminishing returns. For example, for $M = 3$, the required fade margin improves to 6.15 dB, a reduction of 3.6 dB, which isn't as great as the reduction in fade margin due to changing M from 1 to 2.

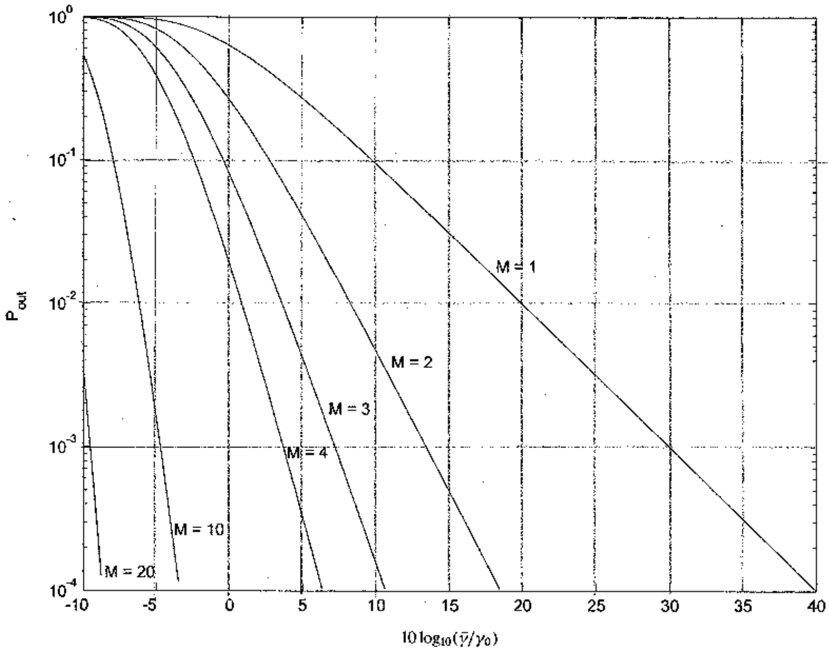

Figure 7.5: P_{out} for maximal-ratio combining with i.i.d. Rayleigh fading.

Figure 3: Goldsmith Figure 7.5, the impact of maximal ratio combining.

2.2.2 Scanning Combining

Selection combining assumes we know all signal amplitudes so that we can take the maximum. Scanning combining is a simplification which says that we only have one receiver, so we can only know the signal to noise ratio on one channel at a time. But we can switch between them when one channel's SNR drops too low. We can often achieve nearly the same results using a scanning combiner as with selection combining.

2.2.3 Equal Gain Combining

Here, we simply co-phase the signals and then add them together. The outage probability improves compared to selection combining. Denoting the SNR of the summed signal as γ_Σ, an analytical expression for the outage probability given Rayleigh fading is [1, p. 216],

$$P\left[\gamma_\Sigma < \gamma\right] = 1 - e^{-2\gamma/\Gamma} - \sqrt{\pi\gamma/\Gamma}e^{-\gamma/\Gamma}\left(1 - 2\mathrm{Q}\left(\sqrt{2\gamma/\Gamma}\right)\right)$$

where $\mathrm{Q}\left(\cdot\right)$ is the tail probability (complementary CDF) of a zero-mean unit-variance Gaussian random variable.

2.2.4 Maximal Ratio Combining

For maximal ratio combining, we still co-phase the signals. But then, we weight the signals according to their SNR. The intuition is that some channels are more reliable than others, so we should "listen" to their signal more than others – just like if you hear something from multiple friends, you probably will not weight each friend equally, because you know who is more reliable than others.

The outage probability improves compared to equal gain combining. Denoting the SNR of the summed signal as γ_Σ, an analytical expression for the outage probability given Rayleigh fading is [1, p. 214],

$$P\left[\gamma_\Sigma < \gamma\right] = 1 - e^{-\gamma/\Gamma} \sum_{k=1}^{M} \frac{(\gamma/\Gamma)^{k-1}}{(k-1)!}.$$

References

[1] A. Goldsmith. *Wireless communications*. Cambridge University Press, 2005.