

## Lecture 13

Today: (1) PAM, QAM, and PSK, (2) Questions

- Reading: Today: Molish 12.1; Tue after break: Goldsmith handout (on Blackboard).
- Exam 2 is on Tuesday Feb 26 at 9:10am - 10:10am.
- Homework 5 due at start of class today.

## 1 PAM, QAM, and PSK Modulation

Last lecture we talked about how a digital transmitter sends one of  $M$  symbols, that is, linear combinations, of a small number  $N$  of orthogonal waveforms. The TX's choice from the  $M$  possible linear combinations lets it send one of  $\log_2 M$  bits per symbol. This section talks specifically about options for linear combinations when we set our the first waveform to a cosine, and the second to a sinusoid.

### 1.1 PAM

Three “types” of modulations only use one (orthogonal) waveform,  $\phi_0(t) = \cos(2\pi f_c t)p(t)$ , where  $p(t)$  is the pulse shape. (It has nothing to be orthogonal to, except for waveforms sent during other symbol periods.) A linear combination, then, is just  $a_{i,0}p(t)$ , for some constant  $a_{i,0}$ . A few different modulations are so named by their choice of  $M$  different choices of  $a_{i,0}$ , for  $i = 0, \dots, M - 1$ .

- On-off-keying (OOK):  $M = 2$ , and we choose  $a_{0,0} = 0$ , and  $a_{1,0} = 1$ . When sending a “0” bit, the transmitter doesn't send anything. It only actually transmits energy when sending a “1” bit.
- Binary PAM, a.k.a. binary phase-shift-keying (BPSK):  $M = 2$ , and we choose  $a_{0,0} = -1$ , and  $a_{1,0} = 1$ . Now, the amplitude of the sinusoid being transmitted is switched from a +1 to a -1, or vice versa, when the bit switches. This is also a phase shift of  $180^\circ$ , which is why it can be called “phase shift” keying.
- $M$ -ary PAM: Binary PAM is extended in  $M$ -ary PAM to include  $M$  (for  $M$  equal to some power of 2) equally spaced amplitudes, centered on zero. So, for example,  $M = 4$  PAM would have  $a_{0,0} = -3$ ,  $a_{1,0} = -1$ ,  $a_{2,0} = +1$ ,  $a_{3,0} = +3$ . In general, for  $M$ -ary PAM,  $a_{i,0} = 2i - M + 1$ .

Note that *differential phase shift keying* (DPSK) is an implementation variant of BPSK, which we will discuss next lecture.

## 1.2 M-ary QAM and PSK

Many types of modulations use the following two orthogonal waveforms:

$$\begin{aligned}\phi_0(t) &= \cos(2\pi f_c t)p(t) \\ \phi_1(t) &= \sin(2\pi f_c t)p(t)\end{aligned}$$

where  $p(t)$ , again, is the pulse shape. These symbols  $\mathbf{a}_i = [a_{i,0}, a_{i,1}]^T$  can then be plotted on a 2-D graph, typically with the  $\phi_0(t)$  amplitude plotted on the horizontal axis, and the  $\phi_1(t)$  amplitude plotted on the vertical axis.

There are two main “types” of modulations which use these two orthogonal waveforms:

- Phase-shift keying (PSK): PSK places all  $\mathbf{a}_i$  uniformly on the unit circle. That is,  $\|\mathbf{a}_i\| = 1$  for all  $i$ . For  $M = 2$ , this is the same as BPSK. For  $M = 4$ , the symbols are spaced every 90 degrees and thus the constellation looks like the four corners of a square, and is also called quadrature phase shift keying (QPSK). For  $M = 8$ , symbols are every 45 degrees apart.
- QAM: Generally, any modulation that uses  $\phi_0(t)$  and  $\phi_1(t)$  above can be considered as QAM. So PSK is a sometimes a subtype of QAM. But QAM is not limited to  $\|\mathbf{a}_i\| = 1$ . For example, square QAM has  $M = 2^{2k}$  for some integer  $k \geq 4$ , where symbols are arranged in a square grid, centered at zero. Note  $M = 4$  square QAM is the same as QPSK. But  $M = 16$  and  $M = 64$  are also common values of  $M$  for square QAM.

We drew in the previous lecture notes five different constellation diagrams of PSK and QAM modulations.

Note that for QPSK or square  $M$ -QAM, you send  $\log_2 M$  bits total per symbol. We can also look at the modulation as sending two independent signals, one on the in-phase (horizontal axis) and one on the quadrature (vertical axis), each sending  $\frac{1}{2} \log_2 M$  bits per symbol.

Note that OQPSK and  $\pi/4$  QPSK are variations on QPSK, that have equivalent fidelity performance and identical bandwidth as QPSK, but they are “constant envelope” modulations, which we will discuss later.

**Bandwidth:** Note that for PAM, PSK, and QAM, assuming SRRC pulse shaping with rolloff factor  $\alpha$ , the null-to-null bandwidth of the signal is  $B = (1 + \alpha)/T_s$ , where  $1/T_s$  is the symbol rate.