ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 14

Today: (1) FSK, (2) Fidelity

- Reading: Today: Mol 11.3, Mol 12.1. Tue: none.
- Exam 2: Solutions.
- Homework 6 due Tuesday at noon.

1 FSK

Binary FSK modulations use the following orthogonal waveforms:

$$\begin{aligned} \phi_0(t) &= \cos\left(2\pi\left[f_c - n\Delta f\right]t\right)p(t) \\ \phi_1(t) &= \cos\left(2\pi\left[f_c + n\Delta f\right]t\right)p(t) \end{aligned}$$

for some positive integer n, where $\Delta f = \frac{1}{4T_s}$. We showed that for $n = 2$, that is, when the frequency difference between ϕ_0 and ϕ_1 ("offset") is $1/T_s$, that the two are orthogonal. Standard binary FSK uses $n = 2$.

Bandwidth: For Binary FSK, when a SRRC pulse is used in $p(t)$, the transmitted signal bandwidth is given by

$$B = 2n\Delta f + (1 + \alpha)R_s$$

where $R_s = R_b = 1/T_b$. So, for Binary FSK,

$$B = 4\frac{1}{4T_s} + (1 + \alpha)R_s = (2 + \alpha)R_s$$

1.1 MSK

You can also show that orthogonality is achieved for any integer n in (1). Since putting the two sinusoids closer together reduces the bandwidth of the signal, this is a good idea for spectral efficiency. When $n = 1$ binary FSK is called *minimum shift keying (MSK)*.

Sidenote: MSK is actually the same as OQPSK with a half-cosine pulse shape. We do not discuss OQPSK until next lecture.

1.2 Receiver Complexity Options

One issue in receivers is the costs of coherent vs. non-coherent reception.

Def'n: *Coherent Reception*
A receiver is coherent if it is phase-synchronous, that is, it estimates and then uses the phase of the incoming signal.

Coherent reception requires a phase-locked loop (PLL) for each different carrier frequency in the signal. This can be a problem for FSK receivers that have high M. Also, accurate phase-locked loops can be difficult to achieve for mobile radios for cases with high Doppler. Non-coherent reception just uses an energy detector (a filter, square, and integrate) on each of the carrier frequencies, and then the bit decision is made by seeing which frequency has the highest energy.

Def'n: *Differential Coding*
A coding scheme is differential if the transmitted symbol k changes compared to the $k-1$st symbol whenever a "1" is to be transmitted, or stays the same as the $k-1$st symbol whenever a "0" is to be transmitted.

Differential reception is another technique to aid receivers which operate without accurate phase. When Doppler is present, and the coherent receiver could end up with a consistent phase error of 180 degrees. In that case, a standard receiver will flip every subsequent bit estimate. A differential receiver looks only the change in phase. In the same case, when the phase is flipped by 180 degrees, one bit error will be made because of the flip, but the subsequent bits will be correct. NOTE: Differential coding can be used for any modulation, not just FSK, although it is typically limited to binary modulations.

2 Fidelity

Now, let's compare the fidelity (probability of bit error and probability of symbol error) across different modulations and receiver complexity options. For the same energy per bit, using a different modulation would result in a different fidelity.

Recall $N = FkT_0B$. We also define $N_0 = FkT_0$, so that $N = N_0B$. The units of N_0 are Watts per Hertz, or equivalently, Joules.

The signal power at the receiver can also be written in terms of the energy per symbol or energy per bit. Since energy is power times time, the energy used to send a symbol duration T_s is $\mathcal{E}_s = ST_s$, where S is the received power P_r, and $\mathcal{E}_s$ has units Joules per symbol. To calculate the energy needed to receive one bit, we calculate $\mathcal{E}_b = ST_s/\log_2 M$ Joules per bit. To shorten this, we define $T_b = T_s/\log_2 M$ and then $\mathcal{E}_b = ST_b$ or $\mathcal{E}_b = S/R$, where $R = 1/T_b$ is the bit rate.

The probability of bit error is a function of the ratio $\mathcal{E}_b/N_0$. For example, for BPSK,

$$P_{e,BPSK} = \mathrm{Q}\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$$

where $\mathrm{Q}(z)$ is the tail probability of a Gaussian r.v., given by

$$\mathrm{Q}(z) = \frac{1}{2}\mathrm{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

Name	P [symbol error]	P [bit error]
BPSK	$= \mathrm{Q}\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$	same
OOK	$= \mathrm{Q}\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$	same
DPSK	$= \frac{1}{2}\exp\left(-\frac{\mathcal{E}_b}{N_0}\right)$	same
M-PAM	$= \frac{2(M-1)}{M}\mathrm{Q}\left(\sqrt{\frac{6\log_2 M}{M^2-1}\frac{\mathcal{E}_b}{N_0}}\right)$	$\approx \frac{1}{\log_2 M} P$ [symbol error]
QPSK		$= \mathrm{Q}\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$
M-PSK	$\leq 2\mathrm{Q}\left(\sqrt{2(\log_2 M)\sin^2(\pi/M)\frac{\mathcal{E}_b}{N_0}}\right)$	$\approx \frac{1}{\log_2 M} P$ [symbol error]
Square M-QAM		$\approx \frac{4}{\log_2 M}\frac{(\sqrt{M}-1)}{\sqrt{M}}\mathrm{Q}\left(\sqrt{\frac{3\log_2 M}{M-1}\frac{\mathcal{E}_b}{N_0}}\right)$
2-non-co-FSK	$= \frac{1}{2}\exp\left[-\frac{\mathcal{E}_b}{2N_0}\right]$	same
M-non-co-FSK	$= \sum_{n=1}^{M-1}\binom{M-1}{n}\frac{(-1)^{n+1}}{n+1}\exp\left[-\frac{n\log_2 M}{n+1}\frac{\mathcal{E}_b}{N_0}\right]$	$= \frac{M/2}{M-1} P$ [symbol error]
2-co-FSK	$= \mathrm{Q}\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$	same
M-co-FSK	$\leq (M-1)\mathrm{Q}\left(\sqrt{\log_2 M\frac{\mathcal{E}_b}{N_0}}\right)$	$= \frac{M/2}{M-1} P$ [symbol error]

Table 1: Summary of probability of bit and symbol error formulas for several modulations.

Put this into Matlab or your calculator, it is a useful function. You might also use a table. The exam 3 will have a plot of the Q function and a table of the Q-inverse function.

Other symbol error rates and bit error rates for a variety of modulations is shown in Table 2.

What are the most important? In my opinion: BPSK, DPSK, QPSK, 16-QAM, 64-QAM, 2-non-co-FSK, 2-co-FSK. For the homework, you will make a table, for each modulation, listing P [bit error] formula, and bandwidth. You will also plot P [bit error] as a function of $\frac{\mathcal{E}_b}{N_0}$. Both will be very useful to have in your portfolio so that you have them available on the exam 2. An example is shown in Figure 1.

3 Link Budgets with Digital Modulation

Link budgeting was discussed in the first part of this course assuming that the required SNR was known. But actually, we deal with a required $\frac{\mathcal{E}_b}{N_0}$. Note

$$\frac{\mathcal{E}_b}{N_0} = \frac{S}{N_0}\frac{1}{R_b} = \frac{S}{N}\frac{B}{R_b}$$

Typically the ratio of bandwidth to bit rate is known. For example, what is $\frac{B}{R_b}$ for SRRC pulse shaping?

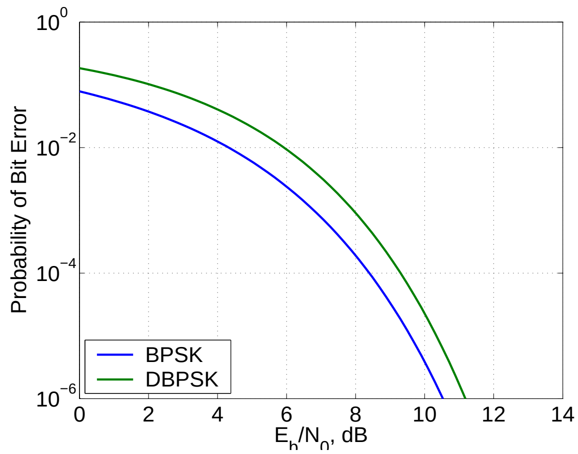

Figure 1: Comparison of probability of bit error for BPSK and Differential BPSK.

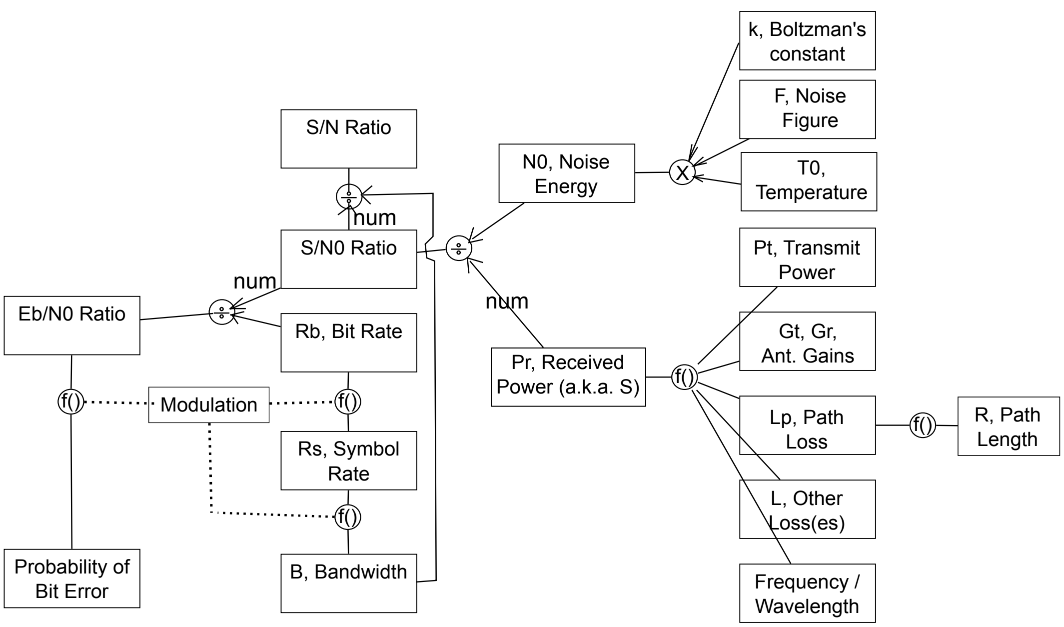

Figure 2: Link Budgeting including Modulation: This relationship graph summarizes the relationships between important variables in the link budget and the choice of modulation.