ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

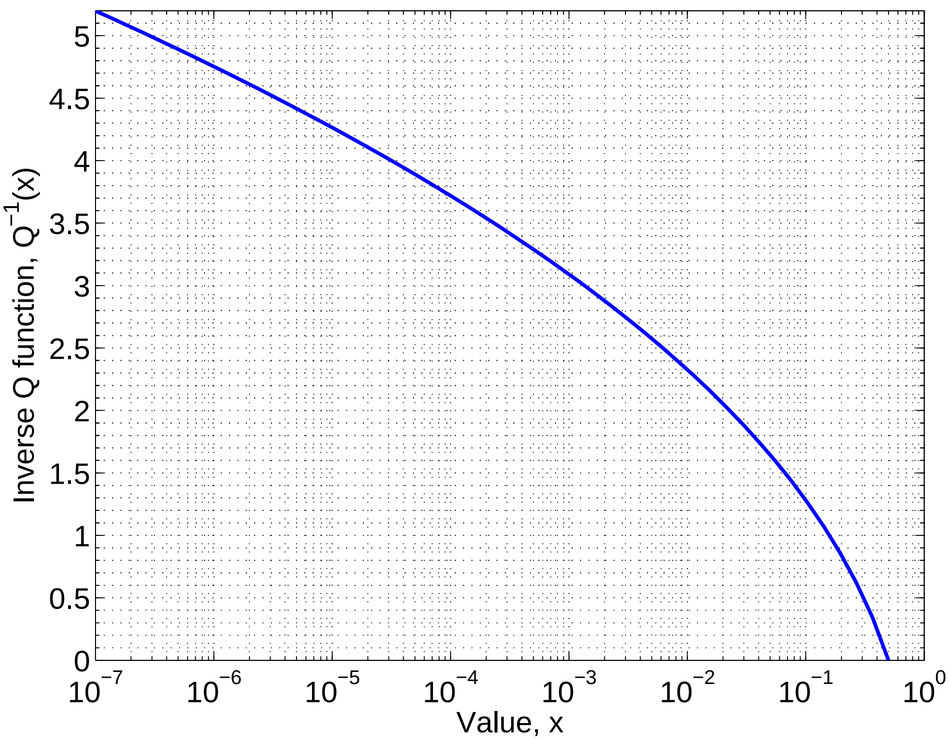
Lecture 15

Today: (1)

- Today: Mol 11.3, 12.1; Thursday: Mol 19.1, 19.2, 19.3, 19.4.
- Homework 6 due Wed at noon.
- Homework 7 still due Tue, Mar 12 at noon.

TABLE OF THE $Q^{-1}(\cdot)$ FUNCTION:

$Q^{-1}(1\times10^{-6}) = 4.7534$	$Q^{-1}(1\times10^{-4}) = 3.719$	$Q^{-1}(1\times10^{-2}) = 2.3263$
$Q^{-1}(1.5\times10^{-6}) = 4.6708$	$Q^{-1}(1.5\times10^{-4}) = 3.6153$	$Q^{-1}(1.5\times10^{-2}) = 2.1701$
$Q^{-1}(2\times10^{-6}) = 4.6114$	$Q^{-1}(2\times10^{-4}) = 3.5401$	$Q^{-1}(2\times10^{-2}) = 2.0537$
$Q^{-1}(3\times10^{-6}) = 4.5264$	$Q^{-1}(3\times10^{-4}) = 3.4316$	$Q^{-1}(3\times10^{-2}) = 1.8808$
$Q^{-1}(4\times10^{-6}) = 4.4652$	$Q^{-1}(4\times10^{-4}) = 3.3528$	$Q^{-1}(4\times10^{-2}) = 1.7507$
$Q^{-1}(5\times10^{-6}) = 4.4172$	$Q^{-1}(5\times10^{-4}) = 3.2905$	$Q^{-1}(5\times10^{-2}) = 1.6449$
$Q^{-1}(6\times10^{-6}) = 4.3776$	$Q^{-1}(6\times10^{-4}) = 3.2389$	$Q^{-1}(6\times10^{-2}) = 1.5548$
$Q^{-1}(7\times10^{-6}) = 4.3439$	$Q^{-1}(7\times10^{-4}) = 3.1947$	$Q^{-1}(7\times10^{-2}) = 1.4758$
$Q^{-1}(8\times10^{-6}) = 4.3145$	$Q^{-1}(8\times10^{-4}) = 3.1559$	$Q^{-1}(8\times10^{-2}) = 1.4051$
$Q^{-1}(9\times10^{-6}) = 4.2884$	$Q^{-1}(9\times10^{-4}) = 3.1214$	$Q^{-1}(9\times10^{-2}) = 1.3408$
$Q^{-1}(1\times10^{-5}) = 4.2649$	$Q^{-1}(1\times10^{-3}) = 3.0902$	$Q^{-1}(1\times10^{-1}) = 1.2816$
$Q^{-1}(1.5\times10^{-5}) = 4.1735$	$Q^{-1}(1.5\times10^{-3}) = 2.9677$	$Q^{-1}(1.5\times10^{-1}) = 1.0364$
$Q^{-1}(2\times10^{-5}) = 4.1075$	$Q^{-1}(2\times10^{-3}) = 2.8782$	$Q^{-1}(2\times10^{-1}) = 0.84162$
$Q^{-1}(3\times10^{-5}) = 4.0128$	$Q^{-1}(3\times10^{-3}) = 2.7478$	$Q^{-1}(3\times10^{-1}) = 0.5244$
$Q^{-1}(4\times10^{-5}) = 3.9444$	$Q^{-1}(4\times10^{-3}) = 2.6521$	$Q^{-1}(4\times10^{-1}) = 0.25335$
$Q^{-1}(5\times10^{-5}) = 3.8906$	$Q^{-1}(5\times10^{-3}) = 2.5758$	$Q^{-1}(5\times10^{-1}) = 0$
$Q^{-1}(6\times10^{-5}) = 3.8461$	$Q^{-1}(6\times10^{-3}) = 2.5121$	
$Q^{-1}(7\times10^{-5}) = 3.8082$	$Q^{-1}(7\times10^{-3}) = 2.4573$	
$Q^{-1}(8\times10^{-5}) = 3.775$	$Q^{-1}(8\times10^{-3}) = 2.4089$	
$Q^{-1}(9\times10^{-5}) = 3.7455$	$Q^{-1}(9\times10^{-3}) = 2.3656$	



Example: Range Comparison
Consider a wireless LAN system at $f_c = 900$ MHz with P_t =1 W, $G_t = 2$, $G_r = 1.6$, a receiver noise figure $F = 8$, a fade margin of 18 dB, SRRC pulse shaping with $\alpha = 0.25$, and path loss given by the log-distance model with $n_p = 3.2$ after a reference distance 1 meter (and free space up to 1 m).

1. Use DPSK with 1 Mbps, with a P[bit error] of 10^{-3} or 10^{-6}.

2. Use 64-QAM with 8 Mbps, with a P[bit error] of 10^{-3} or 10^{-6}.

1 Implementation Costs

1.1 Bandwidth

- For PAM, PSK, and QAM, assuming SRRC pulse shaping with rolloff factor α, the null-to-null bandwidth of the signal is $B = (1 + \alpha)/T_s$, where $1/T_s$ is the symbol rate.

- For binary FSK, as described in the Lecture 14 notes, assuming SRRC pulse shaping with rolloff factor α, the null-to-null bandwidth of the signal is $B = (2 + \alpha)/T_s$.

1.2 Power Amplifiers and Constant Envelope

Recall that the PAM modulation format uses waveforms that are orthogonal at different time periods. For time period k, we can write $\phi_0(t - kT_s) = p(t - kT_s)\cos(2\pi f_c t)$, for a pulse shape $p(t)$ and a carrier frequency f_c. The total transmitted signal is a linear combination of symbols sent at different symbol periods:

$$s(t) = \sum_k a_0^{(k)} p(t - kT_s)\cos(2\pi f_c t) = \cos(2\pi f_c t) \sum_k a_0^{(k)} p(t - kT_s) \quad (1)$$

where $a_0^{(k)}$ is the amplitude of ϕ_0 during symbol k. This cosine term itself has constant power (power itself for a wave is a short time-average of the squared value). So the power at time t is proportional to the square of the $\sum_k a^{(k)} p(t - kT_s)$ term. Consider Figure 1, which shows $s(t)$ for binary PAM when three symbols are sent in a row ($k = 0, 1, 2$). The "envelope" of the signal is the dashed line skirting the maximum of the signal. The "power" is the envelope squared. Note that the power is very close to zero between symbols, a value much smaller than the maximum power.

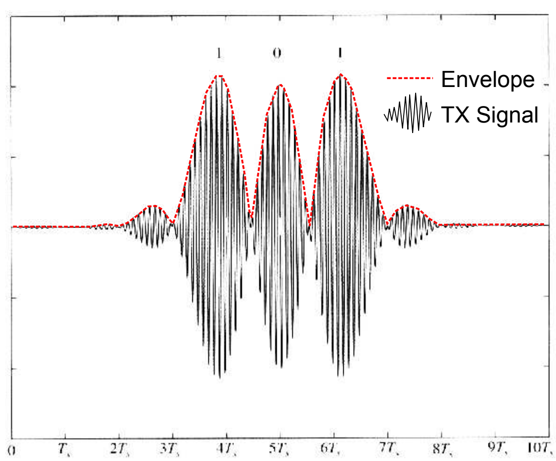


Figure 6.19 Raised cosine filtered ($\alpha = 0.5$) pulses corresponding to 1, 0, 1 data stream for a BPSK signal. Notice that the decision points (at $4T_s$, $5T_s$, $6T_s$) do not always correspond to the maximum values of the RF waveform.

Figure 1: Transmitted signal and signal envelope for an example BPSK signal. Modified from Rappaport Figure 6.19.

A metric to quantify the envelope changes is the peak-to-average power ratio (PAPR). The PAPR is defined as

$$PAPR = \frac{\text{Maximum Power}}{\text{Average Power}}$$

where "Envelope" is the power in transmitted signal $s(t)$. Of course, the maximum is always greater than the average, so the PAPR ≥ 1. But when the max power is not very large compared to the average power, then the PAPR is close to 1, and we say the signal has a constant envelope.

A power amplifier at the transmitter to take the transmitted signal and amplify it to the desired output power (*e.g.*, 500 mW for cellular). Power amplifiers waste some energy, and are rated by their efficiency. Amplifiers that can deal with *any* signal you send to it are about 50% efficient (Class A amplifiers). However, the most efficient (Class C) amplifiers (90% efficient) require a *constant envelope* modulation. Designers thus tend to choose constant envelope modulations for battery-powered transmitters.

In the next section, we compare QPSK (which is not constant envelope) with O-QPSK (which is considered constant envelope).

1.2.1 Offset QPSK

For QPSK, we used orthogonal waveforms:

$$\begin{aligned}\phi_0(t) &= \cos(2\pi f_c t)p(t)\\ \phi_1(t) &= \sin(2\pi f_c t)p(t)\end{aligned}$$

Essentially, there are two 90 degree offset sinusoids multiplying the same pulse shape. After T_s, these sinusoids will multiply $p(t - T_s)$. The effect is similar to Figure 1, but with both cosine and sine terms lined up. Following the notation of (1),

$$s(t) = \cos(2\pi f_c t)\sum_k a_0^{(k)} p(t - kT_s) + \sin(2\pi f_c t)\sum_k a_1^{(k)} p(t - kT_s)$$

We can look at the complex baseband signal as

$$\tilde{s}(t) = \sum_k a_0^{(k)} p(t - kT_s) + i\sum_k a_1^{(k)} p(t - kT_s)$$

The magnitude $|\tilde{s}(t)|$ is the envelope of signal $s(t)$ and $|\tilde{s}(t)|^2$ is its power.

Whenever a_0 switches sign between symbol $k-1$ and symbol k, the linear combination of the pulses will go through zero. When this happens for both a_0 and a_1, the envelope and power go through zero. This means that a linear amplifier (class A) is needed. See Figure 2(b) to see this graphically.

For offset QPSK (OQPSK), we delay the quadrature $T_s/2$ with respect to the in-phase. Now, our orthogonal waveforms are:

$$\begin{aligned}\phi_0(t) &= \cos(2\pi f_c t)p(t)\\ \phi_1(t) &= \sin(2\pi f_c t)p\left(t - \frac{T_s}{2}\right)\end{aligned}$$

Now, the complex baseband signal is,

$$\tilde{s}(t) = \sum_k a_0^{(k)} p(t - kT_s) + i \sum_k a_1^{(k)} p(t - [k + 1/2]T_s)$$

Now, even when both a_0 and a_1 switch signs, we don't see a zero envelope. When a_0 switches sign, the real part will go through zero, but the imaginary part is at a maximum of the current $p(t)$ pulse and is not changing. See Figure 2(d). The imaginary part doesn't change until $T_s/2$ later.

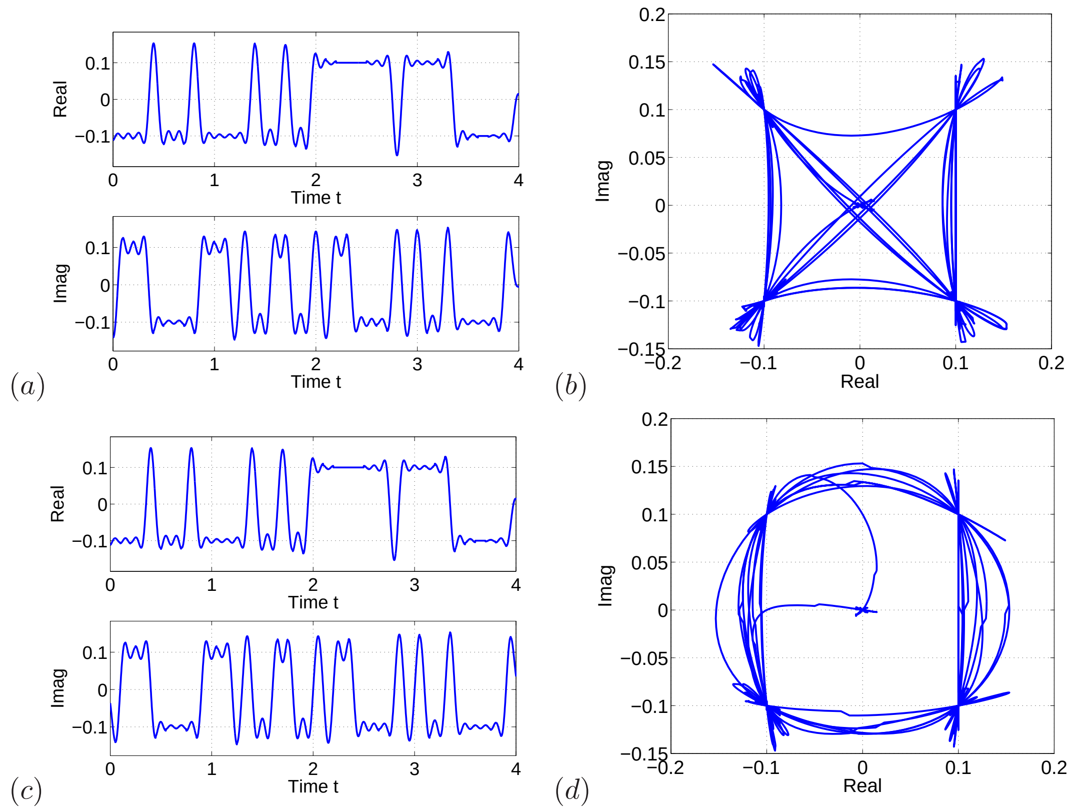


Figure 2: Matlab simulation of (a-b) QPSK and (c-d) O-QPSK, showing the (d) largely constant envelope of OQPSK, compared to (b) that for QPSK.

At the receiver, we just need to delay the sampling on the quadrature half of a sample period with respect to the in-phase signal. The new transmitted signal takes the same bandwidth and average power, and has the same $\frac{\mathcal{E}_b}{N_0}$ vs. probability of bit error performance. However, the envelope $|s(t)|$ is largely constant. See Figure 2 for a comparison of QPSK and OQPSK.

There are some disadvantages of OQPSK: Because the amplitudes of both ϕ_0 and ϕ_1 do not change at the same time, timing synchronization at the receiver can be (somewhat) more difficult [1]. In addition, it is difficult to implement differential decoding with OQPSK.

1.2.2 Other Modulations

What are other modulations that do or do not have a constant envelope?

1. $\pi/4$ QPSK: Alternates between the two QPSK constellations drawn in Lecture 11 Figure 1 (a). Note these are $\pi/4$ radians rotated compared to each other. The envelope will not go as close to zero as QPSK, but is still not considered constant envelope. Used in IS-54.

2. FSK: FSK modulations DO have a constant envelope.

3. M-QAM: Square M-QAM modulations DO NOT have a constant envelope, and are generally even worse than QPSK in that they need a linear amplifier.

1.3 Synchronization

There are two main kinds of synchronization which we require in a receiver:

1. Phase: To multiply by the correct sine or cosine, we need to know the phase of the incoming signal. Typically, we track this with a phase-locked loop (PLL). PLLs can be difficult to implement.

2. Symbol Timing: One must synchronize to the incoming signal to know when to sample the symbol value. Errors in sampling time introduce inter-symbol interference (ISI). Using a lower α makes a receiver more sensitive to symbol synch errors.

In this section, we present two different ways to simplify phase synchronization.

1.3.1 Energy Detection of FSK

FSK reception can be done via a matched filter receiver, in which there is a multiply and integrate with each orthogonal waveform (in this case, sinusoids at different frequencies). This is shown in Figure 3. This is called "coherent reception" of FSK. For binary FSK, there must be 2 PLLs, which have more difficulty, because the received signal contains each frequency sinusoid only half of the time. FSK is popular in inexpensive radios; so two PLLs might be significant enough additional cost to preclude their use.

A non-coherent binary FSK receiver avoids PLLs altogether by simply computing the energy $\mathcal{E}_{f_k}$ at each frequency f_k, for $k = 0, 1$. It decides the bit by seeing which energy, $\mathcal{E}_{f_0}$ or $\mathcal{E}_{f_1}$, is higher.

To calculate the energy, it still must multiply and integrate with a cosine and sine at each frequency. That's because if it just multiplies with the cosine, for example, and the phase of the received signal makes it a sine, then the multiply and integrate will result in zero (sine and

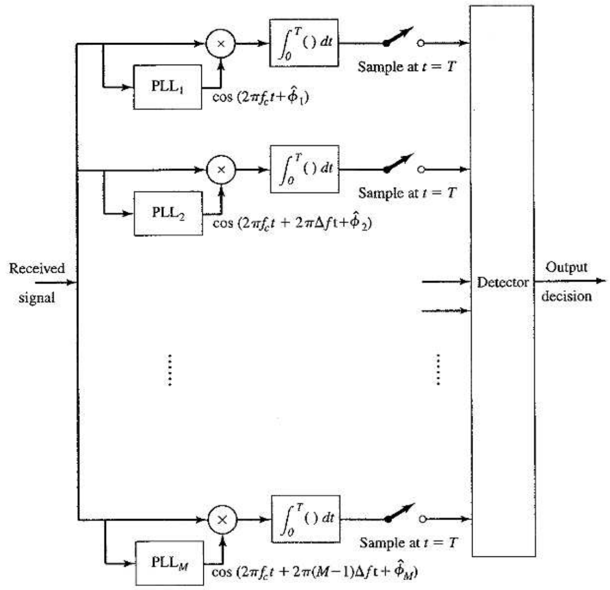


Figure 3: Phase-coherent demodulation of M-ary FSK signals, from Proakis & Salehi [2], Figure 7.46.

cosine are orthogonal, after all). But if you find the energy in the cosine, x_k^I, and the energy in the cosine, x_k^Q, then the total energy at frequency k is

$$\mathcal{E}_{f_k} = (x_k^I)^2 + (x_k^Q)^2$$

is calculated for each frequency f_k, $k = 0, 1$.

The downside of non-coherent reception of FSK is that the probability of bit error increases. The optimal, coherent FSK receiver has performance $\mathrm{Q}\left(\sqrt{\frac{\mathcal{E}_b}{N_0}}\right)$. The non-coherent binary FSK receiver has probability of bit error of $\frac{1}{2}\exp\left[-\frac{\mathcal{E}_b}{2N_0}\right]$. There is about a 1 dB difference between the two – to achieve the same probability of bit error, the non-coherent receiver must have about 1 dB more received power.

1.3.2 Differential PSK

In BPSK, the bit is encoded by the sign of the sinusoid. But the sign is just a 180 degree phase shift in the sinusoid. So when we receive the signal and lock our PLL to the signal, we'll see the phase changing 180 degrees, but we won't know which one is positive and which one is negative. One thing that we can do is to encode the bit using the change of phase. That is, change the phase 180 degrees to indicate a "1" bit, and keep the phase the same to indicate a "0" bit. This is called differential encoding. In the RX, we'll keep track of the phase of the last symbol and compare it to the phase of the current symbol. When that change is closer to 180 degrees than 0 degrees we'll decide that a "1" bit was sent, or alternatively, if that change is closer to 0 degrees than 180

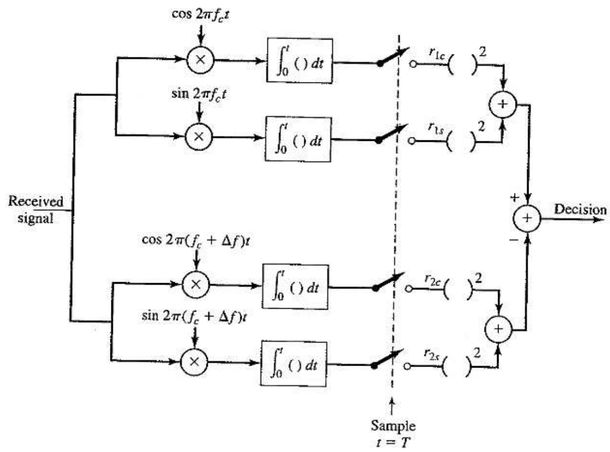


Figure 4: Demodulation and square-law detection of binary FSK signals, from Proakis & Salehi [2], Figure 7.49.

degrees, we'll decide that a "1" bit was sent. This is called differential phase shift keying (DPSK) instead of BPSK. The downside of BPSK is that once we're off by 180 degrees, every bit will be decided in error. Instead, DPSK never becomes 100% wrong, it degrades gracefully with phase (and thus frequency) offset.

The downside of DPSK is that the probability of bit error increases. The optimal BPSK receiver has performance $Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right)$. The DPSK receiver has probability of bit error of $\frac{1}{2}\exp\left[-\frac{\mathcal{E}_b}{N_0}\right]$. There is about a 1 dB difference between the two – to achieve the same probability of bit error, the DPSK receiver must have about 1 dB more received power.

References

[1] K. Pahlavan and A. H. Levesque. *Wireless information networks.* John Wiley & Sons, 2 edition, 2005.

[2] J. G. Proakis and M. Salehi. *Communication System Engineering.* Prentice Hall, 2nd edition, 2002.