ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 16

Today: (1) Multicarrier modulation and OFDM

- HW 7 due Tue after break, March 19, at noon.
- Reading: Today – Molish 19.1-4. Tue after break: Molisch 18.1, 18.2.

1 Multi-carrier Modulation

The basic idea for multi-carrier modulation is to divide the total available bandwidth B into N different subchannels. In each subchannel, we'll use some particular modulation, and send all signals on all subchannels simultaneously. Considering orthogonal waveforms, we'll use both the sin and the cosine function at many different frequencies.

The benefit is that with narrower bandwidth subchannels, we can achieve flat fading on each subchannel. Remember, if the bandwidth is low enough, we don't need an equalizer, because the channel experiences flat fading. (Q: What was the rule given in the Lecture 9 notes given T_s and σ_τ to determine if fading is flat or frequency selective?)

So we will set N high enough so that the symbol rate in each subchannel (which is B/N wide) will be high enough to "qualify" as flat fading. How do you determine the symbol period $\hat{T}_s$ to make the signal fit in B/N bandwidth? (In the Molisch book the "base" multicarrier modulation symbol period is called $\hat{T}_s$). Recall the bandwidth (now B/N) is given by $\frac{1}{\hat{T}_s}(1+\alpha)$ for rolloff factor α so now $\hat{T}_s = \frac{N}{B}(1+\alpha)$. Let's denote T_{orig} as the symbol period that a single carrier system would have used for the original total bandwidth B, that is, $T_{orig} = (1+\alpha)/B$.

If we use $\hat{T}_s \geq 10\sigma_\tau$ then now we can choose N as

$$N \geq 10\sigma_\tau B \frac{1}{1+\alpha}$$

Now, bit errors are not an 'all or nothing' game. In frequency multiplexing, there are N parallel bitstreams, each of rate $\log_2(M)/\hat{T}_s$, where $\log_2(M)/\hat{T}_s$ is the bit rate on each subchannel. As a first order approximation, a subchannel either experiences a high SNR and makes no errors; or is in a severe fade, has a very low SNR, and experiences a BER of 0.5 (the worst bit error rate!). If γ is the probability that a subchannel experiences a severe fade, the overall probability of error will be 0.5γ. The Molisch book has a much more detailed analysis of the probability of error for OFDM in fading channels.

Frequency multiplexing is typically combined with channel coding designed to correct a small percentage of bit errors.

Note that for OFDM in particular, some time T_{cp} at the start of the symbol is called the "cyclic prefix". The Molish book calls the entire symbol period $T_s = \hat{T}_s + T_{cp}$, where $\hat{T}_s$ is the time excluding the cyclic prefix. During the cyclic prefix, we repeat the end of the symbol period, which provides no additional information.

1.1 OFDM

In the previous section we made no restriction on the frequencies of the subcarriers. Well, we know that to have orthogonal sinusoids at different frequencies, we need a particular condition on Δf between the two frequencies.

1.1.1 Orthogonal Waveforms

In FSK, we use a single basis function at each of different frequencies. In QAM, we use two basis functions at the same frequency. Multicarrier modulation (and OFDM is one particular case) is the combination:

$$\begin{aligned}
\phi_{0,I}(t) &= p(t)\cos(2\pi f_c t) \\
\phi_{0,Q}(t) &= p(t)\sin(2\pi f_c t) \\
\phi_{1,I}(t) &= p(t)\cos(2\pi f_c t + 2\pi\Delta f t) \\
\phi_{1,Q}(t) &= p(t)\sin(2\pi f_c t + 2\pi\Delta f t) \\
&\vdots \\
\phi_{N-1,I}(t) &= p(t)\cos(2\pi f_c t + 2\pi(N-1)\Delta f t) \\
\phi_{N-1,Q}(t) &= p(t)\sin(2\pi f_c t + 2\pi(N-1)\Delta f t)
\end{aligned}$$

where $\Delta f = \frac{1}{\hat{T}_s}$, and $p(t)$ is the pulse shape. These are all orthogonal functions! Note we have $2N$ basis functions here. But, we can transmit much more information than possible in M-ary FSK for $M = 2N$, because we use an arbitrary linear combination of the $\phi()$ rather than sending only one orthogonal waveform at a time. In other words, rather than transmitting on one of the M carriers at a given time (like FSK) we transmit information in parallel on all N channels simultaneously. An example state space diagram for $N = 3$ and PAM on each channel is shown in Figure 1.

Note that the subchannels overlap. They are separated by $1/\hat{T}_s$, but the bandwidth of any subcarrier is $(1+\alpha)/\hat{T}_s$, because of the square root raised cosine (SRRC) pulse. Nevertheless, they are orthogonal so they can be separated at the receiver.

1.1.2 Fourier Transform Implementation

The signal on subchannel k might be represented as:

$$x_k(t) = p(t)\left[a_{k,I}(t)\cos(2\pi f_k t) + a_{k,Q}(t)\sin(2\pi f_k t)\right]$$

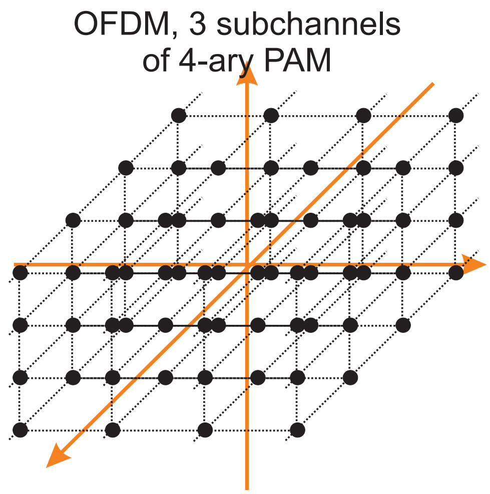

Figure 1: Signal space diagram for $N = 3$ subchannel OFDM with 4-PAM on each channel.

The complex baseband signal of the sum of all N subchannel signals might then be represented as

$$x_l(t) = p(t)\mathbb{R}\left\{\sum_{k=1}^{N}(a_{k,I}(t) + ja_{k,Q}(t))e^{j2\pi k\Delta ft}\right\}$$

$$x_l(t) = p(t)\mathbb{R}\left\{\sum_{k=1}^{N}A_k(t)e^{j2\pi k\Delta ft}\right\} \quad (1)$$

where $A_k(t) = a_{k,I}(t) + ja_{k,Q}(t)$. Does this look like an inverse discrete Fourier transform? If yes, than you can see why it might be possible to use an IFFT and FFT to generate the complex baseband signal.

FFT implementation: There is a particular implementation of the transmitter and receiver that use FFT/IFFT operations. This avoids having N independent transmitter chains and receiver chains. The FFT implementation (and the speed and ease of implementation of the FFT in hardware) is why OFDM is currently so popular.

1.1.3 Cyclic Prefix

However, because the FFT implementation assumes that the data is periodic, and it is not, we must add a cyclic prefix at the start of each block of N symbols. The cyclic prefix is length T_{cp} and is an overhead (doesn't contain data) and thus increases the symbol period. Thus for OFDM, we calculate the OFDM symbol period $T_s = \hat{T}_s + T_{cp}$. This reduces our overall data rate because the symbol period is longer, and no new data is sent during this μT_s cyclic prefix period. The actual data rate will be the total bit rate multiplied by $\frac{N}{N+\mu}$, where $\mu = T_{cp}/T_s$. We determine μ to be longer than the expected delay spread σ_τ. So, $\mu \geq \sigma_\tau/T_s$.

1.1.4 Problems with OFDM

Note that one of the big problems with OFDM is that it requires linear amplifiers, because of its high peak-to-average ratio (PAR). One may prove that the peak to average ratio is approximately N (the number of subcarriers), so for reasonable N, you would need a linear amplifier.

OFDM is also sensitive to timing and frequency offset errors. In a mobile radio system corrupted by Doppler spread, each carrier is spread into the next one, causing the subcarriers to no longer be orthogonal to each other.

1.1.5 Examples

Example: 802.11a
IEEE 802.11a uses OFDM with 52 subcarriers. Four of the subcarriers are reserved for pilot tones, so effectively 48 subcarriers are used for data. Each data subcarrier can be modulated in different ways. One example is to use 16 square QAM on each subcarrier (which is 4 bits per symbol per subcarrier). The symbol rate in 802.11a is 250k/sec. Thus the bit rate is

$$250\times 10^3 \frac{\text{OFDM symbols}}{\text{sec}} 48 \frac{\text{subcarriers}}{\text{OFDM symbol}} 4 \frac{\text{coded bits}}{\text{subcarrier}} = 48 \frac{\text{Mb (coded)}}{\text{sec}}$$

Example: Outdoor OFDM System Design
Design an OFDM system to operate in a 20 MHz bandwidth at 1300 MHz in outdoor environments with large delay spreads of $\sigma_\tau = 4\mu$s. Determine: (1) Does we need multicarrier modulation, and if so, why? (2) The number of subcarriers. (3) The overhead of the cyclic prefix. (4) The symbol period. (5) The data rate of the system assuming $M = 4$ and coding with rate 1/2 is used to correct bit errors.