ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 19

Today: (1) Channel Capacity

- Due: HW 8 – noon today, HW 9 – Apr 2 at **9:10am**, Exam 3 – Apr 4.

1 Shannon-Hartley Bandwidth Efficiency

Ralph V. L. Hartley[1] worked as a researcher in radio telephony for the Western Electric Company, and later, at Bell Laboratories, where he developed relationships useful for determining the capacity of band-limited communication channels [1]. Claude Shannon worked at Bell Labs starting in 1940. In addition to his work on cryptography, Shannon extended the result of Hartley to provide a unifying approach to finding the capacity of digital communications systems in "A mathematical theory of communications" in 1948 [2]. Shannon's result, called the *Shannon-Hartley bound*, provides a fundamental relationship between bandwidth, B (Hz), error-free bit rate, R_b (bits per second), and S/N [2]:

$$R_b \leq C = B \log_2 \left(1 + \frac{S}{N}\right) \tag{1}$$

We can use this to develop a bound on the bandwidth efficiency.

<u>**Def'n:**</u> *Bandwidth efficiency*
The bandwidth efficiency of a digital communication system is the ratio $\eta_B = R_b/B$, which has units of bits per second per Hertz.

The limit on bandwidth efficiency is a direct result of the Shannon-Hartley capacity formula, and is given by,

$$\frac{R_b}{B} \leq \log_2 \left(1 + \frac{R_b}{B}\frac{\mathcal{E}_b}{N_0}\right) \tag{2}$$

Note that this latter expression can't be solved for $\frac{R_b}{B}$ directly – one needs to find a plot of the $\frac{R_b}{B}$ possible for any given $\frac{\mathcal{E}_b}{N_0}$. I have done this using Matlab's "fsolve" function, and a plot of the result is in Figure 1.

Example: Maximum for various $\frac{\mathcal{E}_b}{N_0}$ (dB)
What is the maximum bandwidth efficiency of a link with $\frac{\mathcal{E}_b}{N_0}$(dB) $= 10, 15, 20$? What maximum bit rate can be achieved per 30 kHz of spectrum (one AMPS/USDC channel)? Is this enough for 3G?

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[1]Hartley received the A.B. degree from the University of Utah in 1909.

Solution: Using Figure 1, the $\frac{R_{max}}{B} \leq 6$, 8, and 10 bps/Hz. for $\frac{\mathcal{E}_b}{N_0}(\text{dB}) = 10, 15, 20$. With 30 kHz, multiplying, we have $R_b \leq 180$, 240, 300 kbps. This isn't enough for 3G, which is supposed to achieve up to 14 Mbps.

Shannon's bound is great as engineers, we can come up with a quick answer for what we *cannot* do. But it doesn't necessarily tell us what we *can* achieve. Our P[bit error] equations for particular modulation types are the opposite, giving achievable BERs, but no idea on whether one can do better for the S/N we have.

Note that achieving linear improvements in bps/Hz requires (approximately) linear increases in dB energy. That is, in the example above, one needs to add 5 dB into the link budget for every extra 2 bps/Hz we need. This (5 dB increase) translates to multiplying the linear transmit power by 3.2. It is very difficult to increase bit rate to handle the exponentially increasing user demands simply by increasing power. MIMO is a solution to this limitation – by adding additional channels, via multiple antennas at a transceiver, we can multiply the achievable bandwidth efficiency, as covered in Lecture 20.

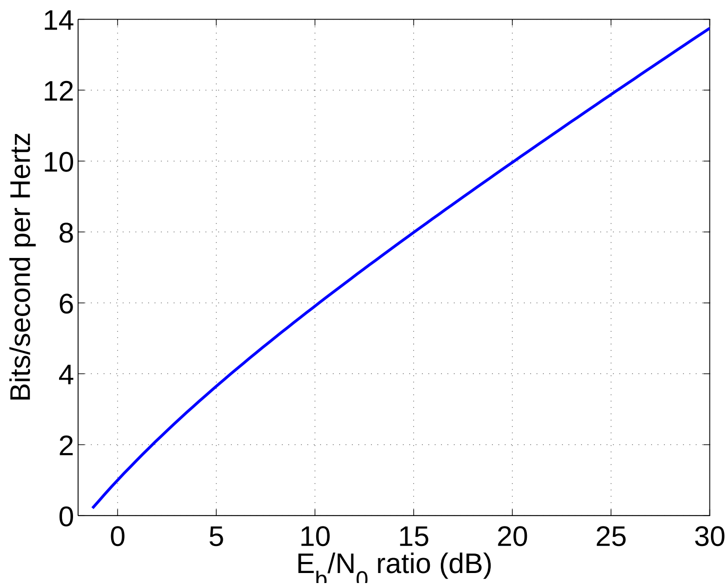

Figure 1: Maximum bandwidth efficiency (bps/Hz) vs. $\frac{\mathcal{E}_b}{N_0}(\text{dB})$.

References

[1] R. V. Hartley. Transmission of information. *AT&T Bell Laboratories Technical Journal*, 1927.

[2] C. Shannon. A mathematical theory of communications. *Bell System Technical Journal*, 27:379–423 and 623–656, 1948. Available at `http://www.cs.bell-labs.com/cm/ms/what/shannonday/paper.html`.