1 Woyach 2006

Discussion questions:

- What experiments were conducted by Woyach et. al.?
- Their work suggests that particular RF sensing systems can be built. In what ways can a transceiver be used as a sensor? What system design work remains?
- What is spatial memory? How is it related to link symmetry?

2 Wideband Channel Modeling [Hashemi 1993]

There are corrections needed to Equations (1) and (2).

Equation (1) should have \(N(t) - 1\) at the top of the sum, rather than \(N(\tau) - 1\):

\[
h(t, \tau) = \sum_{k=0}^{N(t)-1} a_k(t) \delta(\tau - \tau_k(t)) e^{j\theta_k(t)}
\]

\(t\) is “long-term time” or “observation time” and \(\tau\) is “short-term time”. As it says, \(N(t)\) is the number of multipath at the observation time, and \(\{a_k(t)\}, \{\tau_k(t)\}, \text{and} \{\theta_k(t)\}\) are the amplitudes, time delays, and the phases of the multipath components at the current observation time. See Figure 3. The equation (2) should be written as:

\[
h(\tau) = \sum_{k=0}^{N-1} a_k \delta(\tau - \tau_k) e^{j\theta_k}
\] (1)

Essentially, the observation time is removed from the equation.

Hashemi without explanation then renames the time delays \(\tau_k\) to \(t_k\). Use \(t_k = \tau_k\) whenever you need it. Note my equation (1) is \(h(\tau)\), where \(\tau\) is the dummy variable I use for time. One may use any dummy variable for time. Hashemi uses \(t\) in his Equation (2) for the time dummy variable.

Now, \(\tau\) and \(t\) are just dummy variables for the same thing. There is really only one time. If a filter changes over time, it is
no longer time-invariant (TI), and then the channel is not an linear

time-invariant (LTI) filter. The use of both $t$ and $\tau$ is a kludge to

get around the fact that it is not TI. Because the channel changes

relatively slowly (over ms) compared to the time delays of the mul-

tipath (ns), this is generally a good approximation. So at any time

t = t_0$ in ms, we observe the channel impulse reponse, we’ll see some

version of Equation (2). However, a few ms later, at time $t = t_1$,

we’ll observe a different channel impulse reponse, i.e., a different

version of Equation (2).

Discussion items:

1. What is the relationship between complex channel gain; chan-

nel envelope gain; and channel power gain? What is (5) show-

ing? What is an equation for the power gain of the channel?

2. What is the difference between (2), the CIR model for a sta-

tionary channel, and the impulse responses shown in Fig. 1?

Why don’t you see any $\delta$ functions?

3. What is the Fourier transform of (2)? That is, what is the

channel frequency response $H(f)$?

4. What are (a) small-scale, (b) mid-scale, and (c) large-scal-

e variations?

5. What are general characteristics of multipath amplitudes, phases,

and time delays? What statistical models are useful to model

them?

Assuming that phases $\{\theta_k\}$ are i.i.d, and each uniformly dis-

tributed on $[0, 2\pi)$, what is the expected value for the channel power

gain, $R^2$?

$$E[R^2] = E_{\theta_k} \left[ \left| \sum_{k=0}^{N-1} a_k e^{j\theta_k} \right|^2 \right]$$

$$= E_{\theta_k} \left[ \left( \sum_{k=0}^{N-1} a_k e^{j\theta_k} \right) \left( \sum_{l=0}^{N-1} a_l e^{j\theta_l} \right)^* \right]$$

$$= E_{\theta_k} \left[ \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a_k a_l e^{j\theta_k} e^{-j\theta_l} \right]$$

$$= \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} a_k a_l E_{\theta_k, \theta_l} \left[ e^{j(\theta_k - \theta_l)} \right] = \sum_{k=0}^{N-1} a_k^2$$

The last line is because $E_{\theta_k, \theta_l} \left[ e^{j(\theta_k - \theta_l)} \right] = 0$ if $l \neq k$, since they are

i.i.d. uniform from 0 to $2\pi$, which means that the their cosine and

sine have zero mean. When $l = k$, the expected value is

$$E_{\theta_k} \left[ e^{j(\theta_k - \theta_k)} \right] = E_{\theta_k} \left[ e^{j0} \right] = 1.$$