ECE 5325/6325: Wireless Communication Systems
Lecture Notes, Spring 2013

Lecture 21

Today: (1) MIMO

- Exam 3 is one week from today.
- HW 9 due Tuesday at the start of class.
- Reading: today – Haykin/Moher 6.3-6.7, on Canvas. Tue: Abramson paper, Bianchi paper (Sections I and II).

1 MIMO

Multiple-input multiple output (MIMO) is a particular type of space and/or polarization diversity in which both the transmitter and receiver may use multiple antennas.

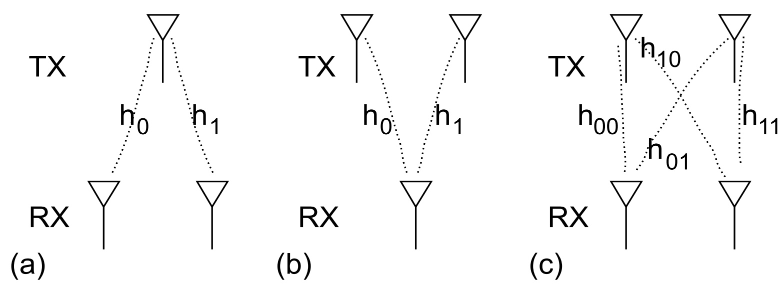

Figure 1: Transmit and receive space diversity schemes: (a) traditional space diversity with receiver combining, called single input multiple output (SIMO); (b) transmit diversity, which may use Alamouti's scheme, called multiple input single output (MISO); (c) 2×2 multiple input multiple output (MIMO).

1.1 Revisit Maximal Ratio Combining

We're going to introduce MIMO by comparing it to maximal ratio combining (MRC) using a receiver with two antennas (space diversity), as shown in Figure 1(a). Let's assume that the transmitter sends symbol voltage s. Assume the channel from TX to RX antenna 0 experiences total channel power gain α_0^2 (or voltage gain α_0), and the channel from TX to RX antenna 1 experiences total channel power gain α_1^2 (or voltage gain α_1).

We said that in MRC, the received signals r_0 and r_1 are multiplied by the square root of the SNRs. Multiplying received signals by a constant doesn't help (it amplifies the noise as much as the signal) so it really matters to multiply them by different numbers. Here, those numbers turn out to be α_0 and α_1.

$$r_{MRC} = \alpha_0 r_0 + \alpha_1 r_1$$

If the transmitted symbol voltage was s, then because the channel voltage gains were α_0 and α_1, we must have that $r_0 = \sqrt{P_t}\alpha_0 s + n_0$ and $r_1 = \sqrt{P_t}\alpha_1 s + n_1$, where n_0, n_1 are the additive noise introduced by the channel. (There would have been phase shifts as well, but remember that co-phasing is done prior to MRC.) So,

$$r_{MRC} = \sqrt{P_t}\left[\alpha_0^2 + \alpha_1^2\right] s + \alpha_0 n_0 + \alpha_1 n_1$$

In the case when we had only one receive antenna, we would have received either r_1 or r_0. In comparison, the noise terms are multiplied by α_0 or α_1, but the signal is multiplied by the sum of $\alpha_0^2 + \alpha_1^2$. If one α_i fades, we don't lose the entire signal s.

1.2 Alamouti code

The MIMO started gaining steam in 1998, from two different results, one from Bell Labs, where they had built an experimental MIMO system they called V-BLAST [4], and a simple transmit diversity scheme from S. M. Alamouti now called the Alamouti scheme [1]. The Alamouti scheme is a simple way to achieve a performance similar to MRC using two transmit antennas, and a single receiver, like the system shown in Figure 1(b). The advantage is that in some cases, the transmitter is more able to have multiple antennas, while the receiver is more limited in size (for example, cellular communications on the downlink).

Alamouti presented a simple scheme that sends two symbols simultaneously, but takes two symbol periods to do so, and over the two transmit antennas. Denote these two symbols s_0 and s_1. The idea is, first transmit s_0 out of antenna 0 and s_1 out of antenna 1. At the receiver, assuming the channels are $h_0 = \alpha_0 e^{j\theta_0}$ and $h_1 \alpha_1 e^{j\theta_1}$, will be

$$r_0 = s_0 \alpha_0 e^{j\theta_0} + s_1 \alpha_1 e^{j\theta_1} \quad (1)$$

Then, during the subsequent symbol period, send $-s_1^*$ out of antenna 0 and s_0^* out of antenna 1, where the superscript $*$ is used to denote complex conjugate. During the second symbol period the receiver will see

$$r_1 = -s_1^* \alpha_0 e^{j\theta_0} + s_0^* \alpha_1 e^{j\theta_1} \quad (2)$$

Note this assumes the channel was the same during the second symbol period as during the first.

The "magic" happens when we combine r_0 and r_1 in the following way to come up with estimates of s_0 and s_1. We form:

$$\begin{aligned}\tilde{s}_0 &= h_0^* r_0 + h_1 r_1^* \\ \tilde{s}_1 &= h_1^* r_0 - h_0 r_1^*\end{aligned}$$

Plugging in for h_0 and h_1,

$$\begin{aligned}\tilde{s}_0 &= \alpha_0 e^{-j\theta_0} r_0 + \alpha_1 e^{j\theta_1} r_1^* \\ \tilde{s}_1 &= \alpha_1 e^{-j\theta_1} r_0 - \alpha_0 e^{j\theta_0} r_1^*\end{aligned}$$

Plugging in for r_0 and r_1 as given in (1) and (2), respectively,

$$\begin{aligned}\tilde{s}_0 &= \alpha_0 e^{-j\theta_0}(s_0\alpha_0 e^{j\theta_0} + s_1\alpha_1 e^{j\theta_1}) + \alpha_1 e^{j\theta_1}(-s_1\alpha_0 e^{-j\theta_0} + s_0\alpha_1 e^{-j\theta_1}) \\ \tilde{s}_1 &= \alpha_1 e^{-j\theta_1}(s_0\alpha_0 e^{j\theta_0} + s_1\alpha_1 e^{j\theta_1}) - \alpha_0 e^{j\theta_0}(-s_1\alpha_0 e^{-j\theta_0} + s_0\alpha_1 e^{-j\theta_1})\end{aligned}$$

Simplifying,

$$\begin{aligned}\tilde{s}_0 &= \alpha_0^2 s_0 + s_1\alpha_0\alpha_1 e^{j(\theta_1-\theta_0)} - s_1\alpha_0\alpha_1 e^{j(\theta_1-\theta_0)} + \alpha_1^2 s_0 \\ \tilde{s}_1 &= \alpha_1^2 s_1 + s_0\alpha_0\alpha_1 e^{j(\theta_0-\theta_1)} - s_0\alpha_0\alpha_1 e^{j(\theta_0-\theta_1)} + \alpha_0^2 s_1\end{aligned}$$

The middle terms cancel out in each case, so finally,

$$\begin{aligned}\tilde{s}_0 &= (\alpha_0^2 + \alpha_1^2)s_0 \\ \tilde{s}_1 &= (\alpha_1^2 + \alpha_0^2)s_1\end{aligned}$$

In short, in two symbol periods, we've managed to convey two symbols of information. Each symbol arrives with approximately the same signal amplitude that we would have had in the maximal ratio combining case.

Notes:

1. This is a two-by one code, that is, it works for two transmit antennas and one receive antenna. This code has been generalized for $n \times m$ MIMO systems, and called "space-time block codes", by Tarokh et. al. [3]. These can send more symbols in less time – in k symbol periods, you can send more than k symbols.

2. If you transmit out of two antennas, you would in general need twice as much power as the receiver diversity case, which had one transmit antenna. So generally we compare the two when using the same total transmit power, *i.e.*, cut the power in half in the transmitter diversity case. The performance is thus 3 dB worse than the receiver MRC diversity case.

3. The Alamouti and space-time block codes are not optimal. Space-time coding is the name of the general area of encoding information the multiple channels. One better-performing scheme is called space-time trellis coding. But the decoding complexity of space-time trellis codes increases exponentially as a function of the spectral efficiency [2, p377] [3].

1.3 MIMO Channel Representation

In general for MIMO, we have multiple (N_t) transmitters and multiple (N_r) receivers. We refer to the system as a (N_t, N_r) or $N_t \times N_r$ MIMO system.

Figure 1(c) shows the channels for a $(2, 2)$ MIMO system. For the channel between transmitter k and receiver i, we denote the "channel voltage gain" as $h_{i,k}$. This gain is a complex number, with real and imaginary parts. Recall that the phase of a multipath component changes with distance, frequency, due to reflections, etc. The channel power gain would be $|h_{i,k}|^2$. The received voltage signal at i, just from transmitter k, is $s_k h_{i,k}$, where s_k is what was transmitted from antenna k.

To keep all these numbers organized, we use vectors and matrices. The transmitted signal from antennas $1, \ldots, N_t$ is denoted $\mathbf{s}$,

$$\mathbf{s} = [s_1, \ldots, s_{N_t}]^T$$

and the channel gain matrix H is given as

$$H = \begin{bmatrix} h_{1,1} & h_{2,1} & \cdots & h_{N_t,1} \\ h_{1,2} & h_{2,2} & \cdots & h_{N_t,2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{1,N_r} & h_{2,N_r} & \cdots & h_{N_t,N_r} \end{bmatrix} \quad (3)$$

Where there are N_r rows each corresponding to the channels measured at each receiver; and N_t columns each corresponding to the channels from each transmitter.

The received signal at receiver i is a linear combination of the s_k for $k = 1, \ldots, N_t$ terms plus noise:

$$x_i = \sum_{k=1}^{N_t} h_{i,k} s_k + w_i$$

where w_i the additive noise term, and $i = 1, \ldots, N_r$. In matrix form, we can rewrite this as:

$$\mathbf{x} = H\mathbf{s} + \mathbf{w}$$

where $\mathbf{x} = [x_1, \ldots, x_{N_r}]^T$ is the received vector and $\mathbf{w} = [w_1, \ldots, w_{N_r}]^T$ is the noise vector.

1.4 Capacity of MIMO Systems

We said in lecture 11 that there is a theoretical limit to the bps per Hz we can achieve on a channel. Using multiple antennas at the TX and RX increases this theoretical limit. We said that the limit on bandwidth efficiency is given as,

$$\frac{R_{max}}{B} = \log_2 (1 + \rho) \quad (4)$$

where R_{max} is the maximum possible bit rate which can be achieved on the channel for given signal to noise ratio ρ and bandwidth B.

In a $N_t \times N_r$ MIMO system with channel matrix H as given in (3), with $N_t \geq N_r$, the new Shannon limit on bps per Hz is [2],

$$\frac{R_{max}}{B} = E\left[\log_2 \left\{\det\left(\mathrm{I}_{N_r} + \rho \frac{1}{N_t} H H^\dagger\right)\right\}\right] \quad (5)$$

where $H^\dagger$ is the complex conjugate of H (I'm copying the notation of the Haykin Moher book), and ρ is the average signal to noise ratio. Here, we assume that each channel is Rayleigh, that is each channel voltage gain $h_{i,k}$ is complex Gaussian, and all channel gains are independent from each other. This is why we need an expected value – the matrix H is filled with random variables.

To get more intuition about the bandwidth efficiency limit, consider that the term $HH^\dagger$ is a Hermitian $N_r \times N_r$ matrix with eigendecomposition $HH^\dagger = U\Lambda U^\dagger$ where U is the matrix of eigenvectors of $HH^\dagger$ and Λ is a diagonal matrix of eigenvalues λ_i for $i = 1, \ldots, N_r$. In this case, we can rewrite (5) as,

$$\frac{R_{max}}{B} = E\left[\sum_{i=1}^{N_r} \log_2\left(1 + \rho\frac{\lambda_i}{N_t}\right)\right] \quad (6)$$

Compared to (4), Equation (6) is a sum of several Shannon capacities – each with effective SNR $\rho\frac{\lambda_i}{N_t}$. Recall this was the formula for $N_t \geq N_r$. For $N_r \geq N_t$, the formula is

$$\frac{R_{max}}{B} = \sum_{i=1}^{N_t} E\left[\log_2\left(1 + \rho\frac{\lambda_i}{N_r}\right)\right] \quad (7)$$

These $\min(N_t, N_r)$ "channels" are called the "eigen-mode channels" of a MIMO system.

In summary, we have created $\min(N_t, N_r)$ eigen-mode channels. Results have shown that the total capacity increases approximately with $\min(N_t, N_r)$. MIMO is so attractive for current and future communication systems because it multiplies the achievable bit rate by this factor of $\min(N_t, N_r)$, without requiring additional bandwidth or signal energy.

References

[1] S. M. Alamouti. A simple transmitter diversity scheme for wireless communications. *IEEE J. Select. Areas Commun.*, 16(8):1451–1458, 1998.

[2] S. Haykin and M. Moher. *Modern Wireless Communications*. Pearson Prentice Hall, 2005.

[3] V. Tarokh, H. Jafarkhani, and A. Calderbank. Space-time block codes from orthogonal designs. *IEEE Transactions on Information Theory*, 45(5):1456–1467, 1999.

[4] P. Wolniansky, G. Foschini, G. Golden, and R. Valenzuela. V-BLAST: an architecture for realizing very high data rates overthe rich-scattering wireless channel. In *URSI International Symposium on Signals, Systems, and Electronics*, pages 295–300, 1998.